A pplication of the polynomial modeling: Determining trajectory, velocity and acceleration of a moving object

P. Hinic\textsuperscript{1}, A. Zagorac\textsuperscript{1}

\textsuperscript{1}Faculty of Electrical Engineering Banja Luka

Abstract. This paper introduces one of the approaches for determining trajectory, velocity and acceleration using polynomial fitting to a collection of measured data points. In analysis that follows we will show, in two different examples with deterministic and stochastic signals, how accurate results can be obtained using the polynomial fitting in given application.

1. INTRODUCTION

Polynomial modeling uses method of least squares and gives us mean values of data. This modeling enables us to overcome fast transients in data which originate from high frequencies. Hence, polynomials obtained using lest squares method generate model for such data. In other words, polynomial modeling of digital data establishes the link between digital and the analog world. This technique enables us to represent digital data in analog form. These analog forms of data are then available for some mathematical operation such as derivation and integration. Suitable example of polynomial modeling is fitting to a collection of data obtained measuring the trajectory of moving object. First derivative of such polynomial describing the trajectory is velocity, while the second derivative is acceleration of a moving object. In analysis that follows, after short introduction to problem of polynomial modeling, we will use method of polynomial fitting to determine trajectory, velocity and acceleration of a moving object. Coordinates of such object are given through sample of deterministic and stochastic functions.

2. POLYNOMIAL FITTING TO A COLLECTION OF DATA POINTS

Suppose that you are faced with N data points, $x_I$. These points can be obtained by measurements of signal’s trajectory. Suppose that obtained data points are fitted with M-th order polynomial and originated at $t=K$. In this case, sum of the square errors between polynomial and given data, becomes

$$J = \sum_{k=0}^{N-1} (x_I - p(t_k - K))^2$$

(1)

Deriving error J at polynomial coefficients (method of least squares) we can generate collection of simultaneous equations. For i-th coefficient, after we have equaled derivation of error J at $a_i$ with zero, we obtain [1]:

$$a_0 \sum_{k=0}^{N-1} (t_k - K)^i + a_1 \sum_{k=0}^{N-1} (t_k - K)^{i+1} + \cdots + a_M \sum_{k=0}^{N-1} (t_k - K)^{i+M} = \sum_{k=0}^{N-1} x_I (t_k - K)^i .$$

(2)

$$i = 0, 1, \ldots, M$$

We can generate different collections of the polynomial, either changing the number of data points in which we do the fitting or changing the order of the polynomial, or both. For $I_k=K+k-p$

3. PRACTICAL EXAMPLES OF DETERMINING THE TRAJECTORY, VELOCITY AND ACCELERATION

Using the theoretical results given in previous section, especially equation (4), we produced program in MATLAB simulating problems for determining trajectory, velocity and acceleration.

3.1. Example 1 - Complex deterministic signals

Suppose that movement of observed object can be described with following parameter equation:

$$x(t) = \cos t + \cos 2t + \cos 3t, \quad y(t) = \sin t + \sin 2t .$$

(5)

Hence, first derivative of signals (5), is

$$x'(t) = -\sin t - 2 \sin 2t - 3 \sin 3t, \quad y'(t) = \cos t + 2 \cos 2t,$$

while the second derivative is given as

$$x''(t) = -\cos t - 4 \cos 2t - 9 \cos 3t, \quad y''(t) = -\sin t - 4 \sin 2t .$$

(7)

Parameter equation (5) can be solved in usual manner, as shown in Figure 1. But if we use different approach: knowing only the data points in trajectory, which we measured, then our problem becomes much complicated. In that case first thing that needs to be done is to model the trajectory (to obtain analogous model of signals), using measured (digital) data. This is done by fitting the given data with M=7-th order polynomial to N=11 points. We concluded that 11 data points are enough for obtaining optimum estimates of functions $x(t)$ and $y(t)$, i.e. functions $a_0=f(t)$, $a_0=f(t)$, Figure 2. Solving the parameter equation $a_0=f(t)$, $a_0=f(t)$, we obtain the trajectory of object, as shown in Figure 3.
Next step is to use fitted data to obtain velocity. That too is done in two different ways, usual way of solving parameter equation (6), Figure 4, and using the polynomial modeling, Figure 6. Velocity, in this scenario, represents the solution of a parameter equation $a_{1x}=f(t)$ and $a_{1y}=f(t)$. Coefficients, $a_{1x}$ and $a_{1y}$, are shown in Figure 5, and represent the estimates of digital data obtained from function (6). Further, we need to determine the acceleration by solving the equation $a_{2x}=f(t)$ and $a_{2y}=f(t)$ (these parameters are represented in Figure 8). Solution of this equation is shown in Figure 9, and is, in fact, desired acceleration. As before, we can see that results obtained in this manner, Figure 9, are same as the results obtained by solving the parameter equation (7), Figure 7. Analyzing obtained results (shown in Fig. 1-9) following conclusions can be made:

- Fitting of N=11 measured data points with M=7-th order polynomial we obtain almost idea estimation $(a_{0x}, a_{0y}, a_{1x}, a_{1y}, a_{2x}, a_{2y})$ of signals (5)-(7) (can bee seen in Figures 2, 5, 8). Values of these coefficients determined from equations:

\[ a_{0x} = -0.0115x_{K+5} + 0.0662x_{K+4} - 0.1267x_{K+3} + \\
0.0115x_{K+2} + 0.3225x_{K+1} + 0.4759x_{K} + 0.3225x_{K-1} + \\
0.0115x_{K-2} - 0.1267x_{K-3} + 0.0662x_{K-4} - 0.0115x_{K-5} \]  
\[ a_{1x} = -0.0091x_{K+5} + 0.0692x_{K+4} - 0.2077x_{K+3} + \\
0.2369x_{K+2} + 0.4177x_{K+1} + 0.5x_{K} - 0.4177x_{K-1} - \\
0.2369x_{K-2} + 0.2077x_{K-3} - 0.0692x_{K-4} + 0.0091x_{K-5} \]

\[ a_{2x} = 0.0157x_{K+5} - 0.0835x_{K+4} + 0.1236x_{K+3} + \\
0.1009x_{K+2} - 0.0732x_{K+1} - 0.1672x_{K} - 0.0732x_{K-1} + \\
0.1009x_{K-2} + 0.1236x_{K-3} - 0.0835x_{K-4} + 0.0157x_{K-5} \]  
Note: Formulas for the given coefficients are the same for the $y$ function and we can easily get them by putting $y$ instead of $x$ in equations (8)-(10).

For better understanding of the polynomial fitting in this example, let us observe following form of the polynomial:

\[ p(I_k - K) = a_0 + a_1(I_k - K) + a_2(I_k - K)^2 + \cdots \]  
(11)

Data points are fitted with polynomial (11). Fitting is done in $x_{K-2}, \ldots, x_{K+2}$ points of every acquired seven-point data set. At $I_k = K$, value of the polynomial equals to the coefficient $p(I_k - K) = a_0$. Coefficient $a_0$, for every $K$, is in fact, estimated value in the middle of the set, i.e. at $t=K$. This way we obtain mean values and estimate in point $t=K$. Doing this for every $K$ (while $K$ is shifting through the coordinate system) we obtain, by means of lest-square method, estimated values for all data from given set.

- In aspect of determining the trajectory, velocity and acceleration, polynomial fitting gives us the same results as the classical way of solving the parameter equations.
3.3. Example 3 – Stochastic signals
Let us observe one stochastic signal (Figure 10) and apply same analysis as for deterministic signals. For the best results, we use the polynomial fitting to N=11 data points of stochastic signal, with M=10-th order of polynomial and obtain optimum estimate, Figure 10. We have determined the trajectory (Figure 11), velocity (Figure 13) and acceleration (Figure 14). Also, we have shown values of the coefficient $a_{1x}$, $a_{1y}$, $a_{2x}$, $a_{2y}$, Figure 12 and 14, respectively. At the same time we cannot compare the first and second derivative of signals with its estimations $a_{1x}$, $a_{1y}$, $a_{2x}$, $a_{2y}$, as we did in previous analysis, because of the nature of the signal.
4. CONCLUSION
Observing the results of simulations in Matlab, Fig. 1-15, following conclusions can be made:

• Fitting to a collection of N=11 data points of a deterministic signal (polynomial order M=7) and N=11 data points of a stochastic signal (but with order M=10 of the polynomial), gives us optimum estimates $a_{ix}$, $a_{iy}$, $i=0,1,2$, for functions $x$, $y$ and their first and second derivative, equations (5)-(7) in example 1, Figures, 2,5,8,10.

• Solution of the parameter equation $a_{ix}=f(t)$, $a_{iy}=f(t)$, $i=0,1,2$, represents the trajectory, velocity and acceleration of a moving object, Figures 3,6,9.

• Comparing the solution of the parameter equations (5)-(7), Figures 1,4,7, with the results obtained applying polynomial fitting (as discussed above), Figures 3, 6, 9, gives the identical results in all cases (trajectory, velocity and acceleration).

5. REFERENCES
[1] P. Hinić, V. Risojević, A. Zagorac, Procesiranje signala, University of Banja Luka, Banja Luka 2000;

APPLICATION OF THE POLYNOMIAL MODELING:
DETERMINING TRAJECTORY, VELOCITY AND ACCELERATION OF A MOVING OBJECT
P. Hinić, A. Zagorac