INTRODUCTION

The problem of the antenna array pattern synthesis with low sidelobes has been investigated by many authors. Generally speaking, methods which were proposed in published papers fall in two main categories and those are analytical and numerical. The analytical methods were developed first, starting with the Dolph's classic paper [1] that showed how the Chebyshev polynomials can be utilized for a pattern synthesis with the uniform sidelobes. This paper has been followed by many others [2,3,4]. Some of these papers [3,4] describe methods that allow radiation patterns to be synthesized with the sidelobe envelope varying with angle. But all of these methods suffer from one common limitation. That is, none of them can solve the problems where the array elements are unequal, nonuniformly spaced, or the array is not linear.

In order to overcome this obstacle, numerical methods based on the adaptive array theory developed by Applebaum [5] were proposed [6,7]. These methods are able to shape radiation pattern of an arbitrary antenna array. Primary idea behind them, is to involve virtual adaptive array that has same elements like array which pattern is being shaped. The main beam is steered in desired direction in the same way it is done in [5] and the sidelobe level is adjusted by including a large number of virtual interfering signals in directions outside of main beam. Power level of those interfering signals is then changed in each iteration with aim to get radiation pattern with desired sidelobe envelope. It is good to emphasize that only in [7], way the interferers power is being changed is presented. This way includes user determined parameter essential for algorithm convergence. Also, unlike in [7], our method does not include matrix inversion and therefore it is computationally more efficient.

PROBLEM FORMULATION

Consider an arbitrary antenna array of N elements. Assume single frequency (CW) signal is incident on the array from angle $\theta$ from boresight. Then, regarding general array theory, this signal will produce vector of the complex voltages on outputs of the array elements:

$$x(\theta) = A(\theta) \cdot u(\theta)$$  \hspace{1cm} (1)

Here, $A(\theta)$ is signal amplitude and $u(\theta)$ is vector of the complex voltages produced by signal of the amplitude equal one. Exact form of $u(\theta)$ is defined in basic array theory [5]. If we define vector of the array weights $w$:

$$w = [w_1, w_2, \ldots, w_N]^T$$  \hspace{1cm} (2)

Then, the complex array response $r(\theta)$ to this CW signal will be:

$$r(\theta) = w^T x(\theta)$$  \hspace{1cm} (3)

The array voltage pattern $p(\theta)$ is defined as magnitude of array response to signal of amplitude equal to unity:

$$p(\theta) = |w^T u(\theta)|$$  \hspace{1cm} (4)

If we consider set of distinct CW signals, arriving from angles $\theta=[\theta_1, \theta_2, \ldots, \theta_M]^T$, with amplitudes $A=[A(\theta_1), A(\theta_2), \ldots, A(\theta_M)]'$, then the vector of array responses to each signal alone,

$$r = [r(\theta_1), r(\theta_2), \ldots, r(\theta_M)]'$$  \hspace{1cm} (5)
can be represented in the linear algebraic form as:
\[ r = Xw \]  
(6)

Here matrix \( X \) is defined as:
\[ X = [x(\theta_1), x(\theta_2), \ldots, x(\theta_M)]_{M \times N} \]  
(7)

What our minimax algorithm is able to do is to find vector of array weights \( w \) which minimizes the maximum element of difference between vector of the array responses \( r \) and vector of desired array responses \( d \):
\[
\min_{w} \{ \max |d - r| \} = \min_{w} \{ \max |d - Xw| \} 
\]  
(8)

In the next section, we will introduce set of arriving angles \( \theta \), set of signal amplitudes \( A \), along with value of vector \( d \) that is suitable for shaping array voltage pattern (4) by our minimax algorithm.

2MINIMAX ALGORITHM PATTERN SYNTHESIS

During research, we have found that the best results in the sidelobe suppression are attained when the pattern is being shaped in a point of the main lobe maximum, on the main lobe borders and outside of the main lobe, that is, in the area of sidelobes. This means that, like in [7], the shape of the main lobe is not constrained, but left to form freely. Corresponding element of vector \( d \) has value 1 in the point of main lobe maximum, and zero elsewhere. This is shown on the figure 1.

Consider now set of CW signals with different levels arriving to the array in the area outside of main lobe, as it is shown on the figure 2. What MM RLS will do is to suppress all maximums of array response to each of these signals to practically same level, as it is also shown on the figure 2. If we recollect way the array voltage pattern has been defined (4), then it is clear that in the area where signals with the higher level are being presented, lower will be the level of the pattern sidelobes and vice versa, as it is shown on the same figure. For example, if we want sidelobes to be 10 dB lower in one area then in some other, then we will present CW signals to the array in that area with 10 dB higher level then in other area.

The same principle is exploited when CW signals on the borders of the main lobe are considered. Their very high level will produce very low level of voltage pattern. In this way, the main lobe beam width is precisely controlled.

3NUMERICAL RESULTS

For the purpose of testing our algorithms performance we selected array shown on figure 3. The array elements are placed on the parabolic curve. All elements are identically oriented and have cosine radiation pattern. Because our algorithm and that proposed in [7] have different concepts in order to compare their performance, we had to adopt two methods.

First, we define sidelobe envelope level and run algorithm proposed in [7]. After that, we run our algorithm with main lobe beam width same as that achieved by algorithm proposed in [7] and same sidelobe envelope shape. The measure of performance is the difference between sidelobe envelope level
attained by two algorithms. Results of this test are shown on figure 4.

![Figure 4. Results of the first test](image)

Second, we define sidelobe envelope shape and main lobe beam width and then run our algorithm. After that, we run algorithm proposed in [7] with sidelobe envelope level achieved by our algorithm. The measure of performance is the difference between main lobe beam width attained by two algorithms. Results of this test are shown on figure 5.

![Figure 5. Results of the second test](image)

As figures indicate, the performance of our algorithm is somewhat better in both cases. This holds not only for the examples presented above, but appears to be a general impression we have had from many examined cases during our research. In our opinion, this result comes from the fact that our algorithm has a mathematically more direct approach to the problem of pattern synthesis.

4CONCLUSION

The minimax approach to the problem of pattern synthesis offers some new possibilities. Designer is now able to define main lobe beam width and sidelobe envelope shape rather than level. This method also does not need any parameters for its convergence and does not include matrix inversion. These characteristics along with a good performance it shows give it advantage in some applications over the methods introduced before.

REFERENCES


Summary: A novel approach to arbitrary array pattern synthesis is proposed. It is based on the minimax algorithm originally developed for mismatched filter design for radars with pulse compression. The proposed approach allows user to define pattern main lobe beam width and sidelobe envelope shape.

ARBITRARY ANTENNA ARRAY PATTERN SYNTHESIS BY THE MINIMAX ALGORITHM

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