APPLICATION OF MULTiresOLUTIONAL BASIS FITTING
RECONSTRUCTION IN IMAGE MAGNIFYING

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I INTRODUCTION

One problem of image interpolation is to magnify a small image without loss in image clarity. This involves interpolating samples using existing samples of the image. If the interpolation method is chosen appropriately, the magnified image has good resolution. There are various traditional methods of interpolation. The wavelet basis reconstruction method proposed by C. Ford and D. M. Etter [1] can be applied as an alternative to other interpolating methods. This multiresolutional basis fitting reconstruction (MBFR) method is very effective. It is suitable for performing reconstruction of signals even from nonuniformly spaced data samples.

This paper is organized as follows. Section 2 briefly describes the method for interpolating one-dimensional signals by fitting data in multiresolutional framework [1]. In Section 3 we have shown that the above method can be extended to two-dimensional signals. Section 4 contains experimental results that we obtained by applying the wavelet basis reconstruction method to resolution enhancement of images. Different wavelet bases are used for 4x magnification of 64x64 picture of Lena and comparison with other methods is made. We conclude this paper in Section 5.

II SIGNAL RECONSTRUCTION BY MULTiresOLUTIONAL FITTING

A function \( f(t) \), which resides in the space \( V_0 \), spanned by the orthogonal set of basic function \( \{\phi(t-n)\} \), can be decomposed to an arbitrary resolution level \( J \) by the representation [2]

\[
f(t) = \sum_{n} c_{j,n} \phi \left( \frac{t}{2^j} - n \right) + \sum_{j=1}^{J} \sum_{n} d_{j,n} \psi \left( \frac{t}{2^j} - n \right) \tag{1}
\]

The multiresolutional decomposition of \( f \) (1) requires a series of nested subspaces of \( V_0 \) given by \( V_{j+1} \subset V_j \), such that \( V_{j+1} = V_j \otimes W_j \). The subspace \( V_j \) is spanned by the scaling basic functions \( \{\phi(u/2^j-n)\} \), while \( W_j \) is spanned by the wavelet basic functions \( \{\psi(u/2^j-n)\} \).

The problem of signal reconstruction by multiresolutional fitting, considered in [1], can be formulated as follows. Find \( M \) uniformly distributed samples of a discrete signal

\[
f = [f(0), f(1), f(2), \ldots, f(M-1)]^T
\]

from only \( P < M \) samples of \( f \) on a nonuniformly sampled subset, indexed by \( \{t_k \in \{0, 1, \ldots, M-1\}, k=0, 1, \ldots, P-1\} \). It is assumed that the available signal

\[
f_s = [f(t_0), f(t_1), f(t_2), \ldots, f(t_{P-1})]^T
\]

is undersampled with respect to the Nyquist frequency of the complete, uniformly sampled signal \( f \).

By viewing the evenly spaced samples on \([0, \ldots, M-1]\) as the subspace \( V_0 \), (1) leads to the following system of equations at any resolution level \( J \geq 1 \):

\[
f(t_0) = \sum_{n} c_{j,0} \phi \left( \frac{t_0}{2^j} - n \right) + \sum_{j=1}^{J} \sum_{n} d_{j,n} \psi \left( \frac{t_0}{2^j} - n \right)
\]

\[
f(t_1) = \sum_{n} c_{j,0} \phi \left( \frac{t_1}{2^j} - n \right) + \sum_{j=1}^{J} \sum_{n} d_{j,n} \psi \left( \frac{t_1}{2^j} - n \right)
\]

\[
\vdots
\]

\[
f(t_{P-1}) = \sum_{n} c_{j,0} \phi \left( \frac{t_{P-1}}{2^j} - n \right) + \sum_{j=1}^{J} \sum_{n} d_{j,n} \psi \left( \frac{t_{P-1}}{2^j} - n \right)
\]

A matrix form of the system is

\[
f_s = G_j^T c_j + \sum_{j=1}^{J} H_j d_j \tag{2}
\]

where \( G_j \) is a matrix of shifts of the scaling function samples at level \( J \) associated with the time index \( t_k \) of each sample, and each \( H_j \) is the matrix of the shifts of the wavelet at each level \( j \). If the signal \( f(t) \) is approximated by its low-frequency components represented by the scaling function in (1) (temporarily ignoring the high-frequency terms), the system (2) can be viewed as

\[
f_0 = G_j \hat{c}_j \tag{4}
\]
where $G_J$ is the matrix of shifts of the scaling function at level $J$ at all integer shifts $[0, 1, \ldots, M-1]$. The error signal $e_0$ at each available sample can be viewed as

$$
e_0 = f_x - \hat{f}_x |_{x = t_0, k = 0, 1, \ldots, P-1} = f_x - G'_J \hat{c}_J = H'_J \hat{d}_J. \quad (5)$$

The error signal $e_0$ can be approximated by its next-finer frequency components, represented by the lowest frequency band of the wavelet portion of (1). The system (5) provides an estimate of the coefficients $d_J$, denoted by $\hat{d}_J$. The first refinement of the original function estimate $\hat{f}_1$ can be computed at every point on the even grid from

$$\hat{f}_1 = G'_J \hat{c}_J + H'_J \hat{d}_J$$

where $H'_J$ is the matrix of shifts of the wavelet at level $J$ at all integer shifts $[0, 1, \ldots, M-1]$.

The same procedure can be repeated to find another refinement of the function. The error signal $e_1$ can be viewed as

$$
e_1 = f_x - \hat{f}_1 |_{x = t_1, k = 0, 1, \ldots, P-1} = f_x - G'_J \hat{c}_J - H'_J \hat{d}_J = H'_J \hat{d}_{j-1} = H'_J \hat{d}_{j-1}. \quad (6)$$

This system can be solved in a least-square sense to find $\hat{d}_{j-1}$. Then $\hat{f}_2$ is

$$\hat{f}_2 = G'_J \hat{c}_J + H'_J \hat{d}_J + H'_{j-1} \hat{d}_{j-1}.$$

The process continues until the finest possible resolution level has been reached. That is when aggregate number of coefficients $\hat{c}_J, \hat{d}_J, \hat{d}_{j-1}, \ldots$ is less than or equal to the number of available samples of the signal.

III IMPLEMENTATION

To magnify a small image we actually interpolate samples between some known samples and in that way the image becomes larger. We have used the method described in previous section to perform this interpolation. The known samples of image are actually obtained by uniformly subsampling some larger image with 2x size. Thus we obtain the samples $f(1), f(3), f(5), \ldots$ and setting $t_0 = 1, t_1 = 3, t_2 = 5, \ldots$ we should find $f(2), f(4), f(6), \ldots$ This method is applied on every row first. So from 64x64 image we have obtained 64x128 image. Then process could be repeated again on the resulting image so we can obtain 4x magnification with 256x256 image. In this process, different wavelet bases could be used to obtain different image quality. All results are obtained with Matlab. Lena is taken as the testing image. Results are shown in next section.

IV EXPERIMENTAL RESULTS

This section contains experimental results obtained with different wavelet basis for illustration of proposed method. Magnified Lena images using wavelet transform extrema extrapolation and bilinear interpolation are also given for comparison. The original 64x64 image is shown in Fig. 1. Magnified 256x256 images using Haar and Daubechies wavelet basis can be seen in Fig. 2. Fig. 3 depicts a magnified 256x256 image obtained by the method proposed in [3]. The magnification performed by bilinear interpolation is shown in Fig. 4.
V SUMMARY

Experimental results show that multiresolutional basis fitting reconstruction method can be implemented for magnifying images but processing time is long. As expected, processing time is longer for longer wavelet bases. From previous section one can see that blocking effects are noticeable when Haar base is used (Fig. 3a). When Daubechies bases are used these effects are less noticeable but there are some blurring effects in the resulting images. The best result is obtained with implementation of db3 wavelet base. In this case the optimal tradeoff between duration of wavelet base (time resolution) and its approximation order (number of vanishing moments) is reached. It should be mentioned that we didn't implement any other technique for image sharpening. Otherwise, resulting images could look better. Pictures obtained with MBFR method are comparable with those obtained using other methods. Further improvements could be achieved in research of more efficient algorithms that would decrease processing time.

REFERENCES


Abstract: The wavelet basis method for reconstruction of nonuniformly sampled signals due to C. Ford and D. M. Etter is applied to resolution enhancement of images. The size of an image is doubled by interpolating a new sample between every two samples of the image in horizontal and vertical direction. Larger degrees of magnification are obtained by iterating this procedure.
The effectiveness of the proposed method using different wavelet bases is investigated. The comparison between proposed method and some other known methods shows that the quality of images is comparable from perceptual aspect. Experimental results are presented to support this claim.