# SAMPLING RATE CONVERSION FOR ARBITRARY RATIO USING TRANSPOSED LINEAR INTERPOLATION AND CIC FILTER


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## 1. INTRODUCTION

In multistandard receivers, the hardware should be configurable or programmable for the reception of different types of signals having different symbol rates. After the AD conversion, utilizing commonly the delta-sigma AD-conversion principle and high oversampling ratio, the sampling rate is reduced to be a low integer multiple of the symbol rate. In this decimation, the desired channel is preserved and other channels and noise are attenuated. The problem is in that the needed decimation factor can be a difficult fractional number or even an irrational number and, for instance, FIR filters used for integer or fractional decimation cannot be efficiently utilized. Decimation factor $R$ in general form is given with

$$R = R_{int} + e = R_{int} b \ , \qquad (1)$$

where $R_{int}$ represents its integer part, and $b$ is non-integer decimation term. Another problem is that there can be disturbing channels that are much stronger (e.g. 80-100 dB) than the desired channel. Therefore, the frequency bands that cause aliasing in decimation should have good attenuation. In addition to these requirements, the overall implementation should be simple because this decimation filter is used in the digital front-end of mobile receivers where the sampling rate is high. [1]

In [2] the decimator structure that fulfills mentioned requirements is presented. It contains cascaded integrator and comb (CIC) filter and linear interpolation. The proposed decimator structure has simple implementation structure and low computational requirements. However, the anti-aliasing and anti-imaging properties of the proposed decimator are restricted to the narrowband input signal. This is mainly because the zeros of the linear interpolator are not clustered about multiples of its output sampling frequency where the spectral images appears. Further, zeros of linear interpolator and repeated passbands of CIC filter are not clustered at the same points on frequency grid thus the repeated passbands of the CIC filter are not attenuated. This is avoided if so called transposed linear interpolation (TLI) is used. The transposed Farrow structure is based on polynomial interpolation with polynomial pieces of length equal to the output sampling period, see [3] for details. Transposed linear interpolator is based on the same principle. Its impulse response is given by two linear pieces of length equal to the output sampling period of linear interpolator. In this way the zeros in frequency response are clustered around spectral images that are positioned around multiples of the output sampling rate and better anti-aliasing and anti-imaging properties are achieved. Therefore in this paper we present efficient decimator structure based on TLI and CIC filter, that has good anti-aliasing and anti-imaging properties.

## 2. BUILDING BLOCKS OF DECIMATOR

Cascaded integrator-comb (CIC) filters are commonly used for decimation and interpolation by integer ratio providing efficient anti-image and anti-alias filtering [4]. These filters have a simple regular structure without multipliers. CIC decimation filter (see **Fig. 1**) consists of $K$ cascaded digital integrator stages operating at high input data rate $F_{in}$, followed by $K$ cascaded comb or differentiator stages operating at low sampling rate $F_{in} / R_{int}$, see **Fig. 2**. Its frequency response is given by

$$H_{CIC}\left(e^{j2pf/F_{in}}\right) = e^{-jpKf(R-1)/F_{in}} \left(\frac{\sin(pR_{int} f / F_{in})}{R_{int}\sin(pf / F_{in})}\right)^K , \qquad (2)$$

where $f/F_{in}$ is the normalized input frequency to CIC filter. Note that the frequency response of CIC filter is periodic with period equal to the input sampling rate.

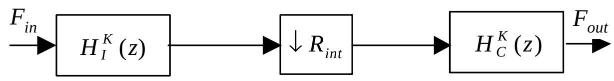


**Fig. 1.** CIC decimation filter. $R_{int}$ is an integer decimation factor and $K$ is the number of cascaded integrator and comb stages.

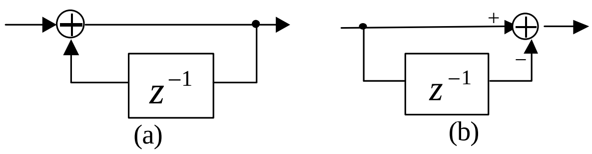


**Fig. 2.** (a) Integrator with the transfer function $H_I^K(z)$ for $K = 1$. (b) Comb filter with the transfer function $H_C^K(z)$ for $K = 1$.

When the decimation factor is an irrational number, the filters intended for integer or fractional decimation can not be directly used. One solution is to use polynomial-based interpolation filters [5]. Among them, linear interpolation filters have a simple implementation structure, only one multiplication is needed. Because interpolation is basically a reconstruction problem, polynomial-based interpolation can be analyzed using the hybrid analog/digital model shown in **Fig. 3**. In this model, the interpolated output samples $y(l)$ are obtained by sampling the reconstructed signal $y_a(t)$ at the time instants $t = (n_l + m_l)\, T_{in}$. Here $n_l$ is any integer, $m_l \in [0,1)$ is the adjustable fractional interval, and $T_{in}$ is the sampling interval of the input signal $x(n)$. The digital implementation of the linear interpolation, which needs only one multiplication, can be based on the following equation:

$$y(l) = x(n_l) + \left[x(n_l+1) - x(n_l)\right]\mu_l . \quad (3)$$

Another implementation possibility in the form of modified Farrow structure is given below

$$y(l) = \frac{1}{2}\left\{\left[x(n_l+1) - x(n_l)\right](2\mu_l - 1) + \left[x(n_l+1) + x(n_l)\right]\right\}. \quad (4)$$

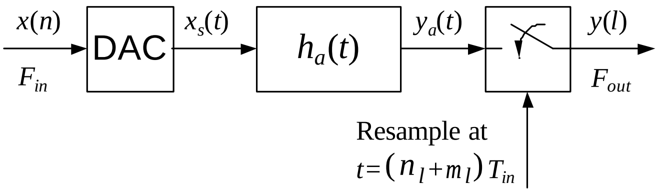


**Fig. 3.** The hybrid analog/digital model for the linear interpolation filter.

For linear interpolation, the impulse response of the reconstruction filter $h_a(t)$ is a triangular function given in **Fig. 4**, and thus, its frequency response is given by

$$H_a(f) = \left(\frac{\sin(\pi f / F_{in})}{\pi f / F_{in}}\right)^2 . \quad (5)$$

It is clear that linear interpolation offers a good attenuation only in the vicinity of the frequencies $nF_{in}$ for $n = 1, 2...$ Therefore, it is important to have a narrowband signal, that is, a high sampling frequency at the input of the linear interpolation filter.

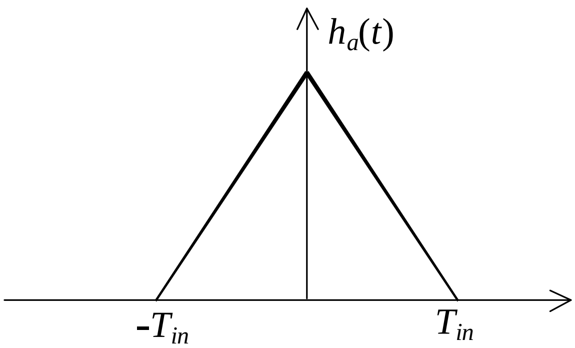


**Fig. 4.** Impulse response of linear interpolator.

Using principle and derivation of transposed Farrow structure given in [3] we propose the transposed linear interpolator principle. The main difference is in length of the linear pieces in impulse response. New length is equal to the output sampling period of linear interpolator $T_{out}$. Thus the frequency response becomes

$$H_a(f) = \left(\frac{\sin(\pi f / F_{out})}{\pi f / F_{out}}\right)^2 . \quad (6)$$

Obviously the zeros are clustered around the multiples of output sampling rate $nF_{out}$ for $n = 1, 2$...The images in spectrum are attenuated effectively. Derivation of implementation structure is not straightforward, and our result is based on theory given in [3]. The starting point is equations given below with $N$=2 and $M$=1,

$$y(l) = \sum_{k=ll}^{ul}\sum_{m=0}^{M} x(k)c_m(i(l,k))(2\mu_k - 1)^m \quad (7)$$

with

$$ul = \left\lfloor (l + \frac{N}{2})\frac{1}{R} \right\rfloor , \quad (8)$$

$$ll = \begin{cases} {(l-N/2)}/{R} + 1 & \text{if } {(l-N/2)}/{R} \in Z \\ \left\lceil {(l-N/2)}/{R} \right\rceil & \text{else,} \end{cases} \quad (9)$$

$$i(l,k) = l - \left\lfloor {k}/{R} \right\rfloor \in \left[ {N}/{2}, {N}/{2} - 1 \right] \quad (10)$$

$$\mu_k = {k}/{R} - \left\lfloor {k}/{R} \right\rfloor . \quad (11)$$

Here $N$ represents the length of the impulse response given in $T_{out}$, while $M$ is degree of polynomial and linear interpolation corresponds to the first degree polynomial. In **Fig. 5** we present transposed linear decimation structure, with coefficients $c_1(-1)$=0.5, $c_0(-1)$=–0.5, $c_1(0)$=0.5, and $c_0(0)$=0.5. The implementation is further simplified by scaling coefficients by 0.5. In this way there is no need for multiplications just output of the structure is shifted by one to the right as that is equal to division by two.

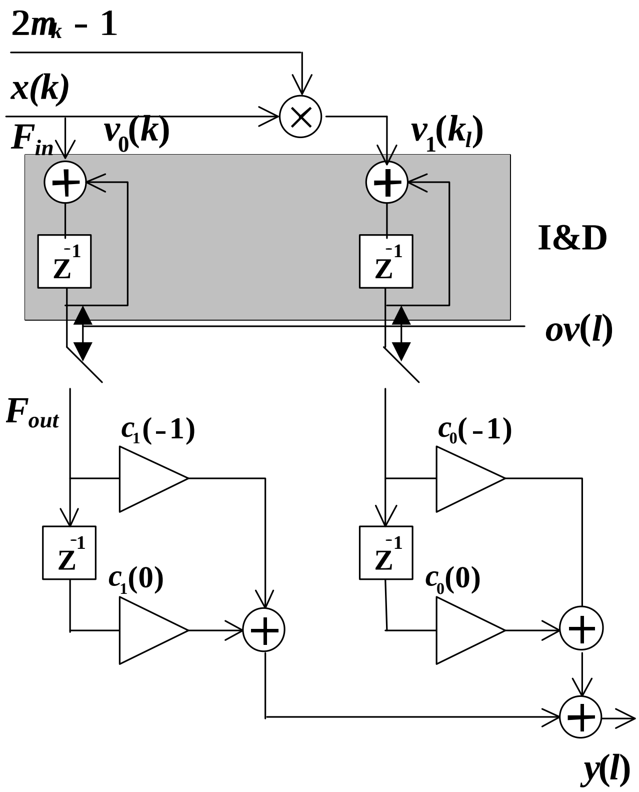


**Fig. 5.** Transposed modified linear interpolation.

The signal $ov(l)$ in **Fig. 5** indicates an overflow of the accumulator, it is set when

$$m_{k+1} \leq m_k \,. \tag{12}$$

When $ov(l)$ is set a new output sample is fed into the output tap delay line followed by a reset of the integrator and dump (I&D) block.

## 3. PROPOSED DECIMATOR STRUCTURE

The proposed decimator structure consists of transposed linear interpolation, denoted by TLI in **Fig. 6** and CIC filter. The structure exploits the benefits of placing the non-integer decimation at the beginning of the decimation chain. Those benefits are low complexity due to the relaxed requirements and LTI works at fixed input rate. Using given order of LTI and CIC, the zeros of LTI are clustered at the same position as repeated passbands of CIC filter that has periodic frequency response.

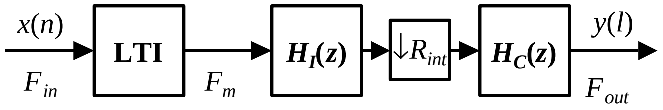


**Fig. 6.** Proposed structure for non-integer decimation.

The overall frequency response of the decimation filter structure is a product of the frequency responses of CIC filter and transposed linear interpolation filter. Note that the former response is a periodical whereas the latter is not. Consequently, the overall zero-phase frequency response of the proposed decimation filter, relative to the input sampling frequency, is given by

$$H_T(w) = H_{CIC}(w) H_a\left(wF_{in}/2p\right) = \left(\frac{\sin\left(\frac{wR_{int}}{2b}\right)}{R_{int}\sin\left(\frac{w}{2b}\right)}\right)^K \left(\frac{\sin\left(\frac{w}{2b}\right)}{\frac{wb}{2}}\right)^2 . \tag{13}$$

where $w = 2p\,f/F_{in} = 2p\,f/(RF_{out})$.

As an example we set arbitrary requirements, same as in [2]. The bandwidth of the input signal is $f_p=0.001F_{in}$ and decimation factor $R=34^1/_{34}$. It is required that the frequency bands that cause aliasing to the frequency band of the input signal are attenuated at least by $A_s$=80dB and the passband distortion is less than $d_p$=0.01 (0.086 dB). These requirements are met by a proposed type of decimation filter having CIC filter of order $K$=3. The overall frequency response of the proposed decimation filter, is shown in **Fig. 7**.

## 4. CONCLUSIONS

We have presented new decimator structure for non-integer sampling rate conversion in multistandard radio receivers. The structure consists of transposed linear interpolator and CIC filter. The main advantage of proposed structure is in that its zeros in frequency response are clustered around multiples of the output sampling rate thus the images in spectra are attenuated. This structure is relatively simple to implement and it has low power consumption as it requires only one multiplier.

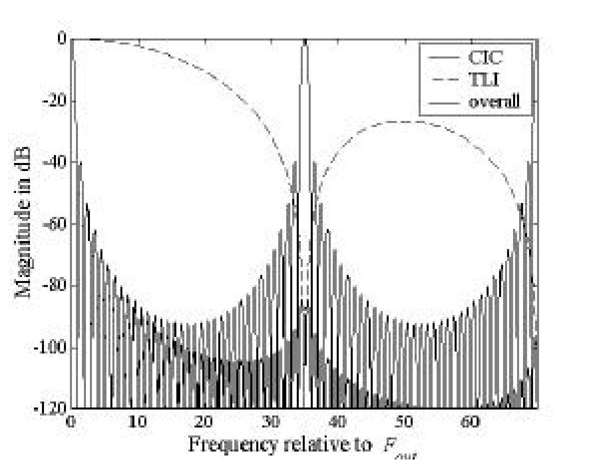


**Fig. 7.** Frequency response of the CIC decimation filter, TLI filter, and the overall response.

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**Sadrzaj:** U ovom radu je prikazana specijalna struktura za promenu frekvencije odabiranja (decimaciju) za radio prijemnike za vise standarda. Struktura se sastoji od transponovanog linearnog interpolacionog filtra i CIC (cascaded integrator-comb) filtra. Glavna prednost predlozene strukture je u tome sto su nule frekevencijskog odziva tacno na mestima gde se javljaju slike u spektru, koje nastaju kao posledica promene ucestanosti odabiranja. Ovo se postize ako je duzina polinomijalnih delova impulsnog odziva filtra za linearnu interpolaciju jednaka izlaznom periodu odabiranja.


**PROMENA UCESTANOSTI ODABIRANJA ZA PROIZVOLJAN FAKTOR UZ UPOTREBU TRANSPONOVANE LINEARNE INTERPOLACIJE I CIC FILTRA, Djordje Babic, Marku Renfors**