DESIGN AND IMPLEMENTATION OF VARIABLE (PROGRAMMABLE) FIR FILTERS

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Abstract- In many filtering applications there is a need for digital filters with variable frequency domain characteristics. This paper gives an overview of an effective design technique of variable filters presented in the literature. In this design technique, multidimensional polynomial functions of spectral parameters are used to approximate the coefficients of the variable FIR filter. The design technique consists of two steps. First, a set of constant FIR filters corresponding to sampled values of spectral parameters is designed by using the known FIR filter design methods. Second, the optimal polynomial coefficients are found using the linear optimization method.

This paper also investigates possible implementation form for this type of variable filters. For the discussed structure the desired frequency domain characteristics are tuned by changing only one spectral parameter associated with it, while the coefficients of the fixed filters are kept unchanged during operation.

1. INTRODUCTION

In many filtering applications, digital filters are required to change their frequency-domain characteristics as different characteristics are needed. The digital filters with variable frequency domain characteristics are referred to as variable filters. Recently, the design and implementations of variable filters have received considerable attention [1]-[5]. One example application area is Software Radio receiver, where a single hardware solution should be adaptable to different system standards by changing software [1].

There are several design methods of variable filters. Many of them are based on a frequency transformation. However, the method which is overviewed here is based on the following. The idea is to determine the coefficients of the transfer function of variable filter as multidimensional function of several spectral parameters [2]. The spectral parameters are, for example, fractional delay, cutoff frequency, passband width etc. In order to do this, a set of optimal constant filters corresponding to some sampled values of the spectral parameters is pre-designed. The resulting coefficients of the constant filters are approximated by using multidimensional functions. The multidimensional functions can be approximated by multidimensional polynomial function, thus polynomial functions are used to approximate the coefficients. This, principle insures efficient implementation of the variable filters. The special implementation form in the case of 1-D polynomial approximation of the filter coefficients is known, so-called, Farrow structure [4]. The design technique of variable filters has two steps:

i) design a set of constant filters, that correspond to specific (sampled) values of spectral parameters,

ii) determine filter coefficients as multidimensional polynomials.

This paper overviews the design technique that solves problem of nonlinear optimization which is required above. This method is presented in [2]. However, this paper considers only variable FIR filters. The design technique of [2] also determines the variable filter coefficients as multidimensional polynomials of spectral parameters. However, this technique includes a linear approximation method for multidimensional polynomials.

This paper also examines efficient implementation form suitable for the case where filter coefficients are represented as multidimensional polynomials. The structure is extension of the Farrow structure [4] to multidimensional polynomials. The Farrow structure is efficient implementation form for polynomial-based filters, where filter coefficients are approximated with 1-D polynomials of certain single spectral parameter. The main advantage of the mention type of structure lies in the fact that it consists of fixed finite-impulse response (FIR) subfilters and multidimensional polynomial network of multipliers. There is only one changeable parameter connected with a single tunable spectral property. In order to tune the spectral property (e.g. fractional delay or magnitude response) of the variable filter during operation, it is required to change only one parameter, while coefficients of the fixed FIR subfilters remain unchanged. Besides this, the control of the programmable spectral parameter is easier during the operation than in the corresponding coefficient memory implementations. Furthermore, the resolution of the control parameter is limited only by the precision of arithmetic used and not by the size of the memory. These characteristics of the implementation structure make it a very attractive form to be implemented using a VLSI circuit or a signal processor [5].

Finally, an illustrative example of variable filter is shown. The example is adopted from [3]. In this example, the design and implementation methods are given and explained for the variable FIR filter with programmable fractional delay and cutoff frequency.

2. DESIGN OF VARIABLE FILTERS

This section gives explanation of the design method of variable filters that is [2]. However, in this paper, this technique is applied to the case of variable FIR filters. It is assumed that the ideal variable filter frequency response is

\[ H_f(\omega, \Psi_1, \Psi_2, \ldots, \Psi_K) = M_f(\omega, \Psi_1, \Psi_2, \ldots, \Psi_K) e^{j\phi(\omega)}, \quad (1) \]

where \( M_f(\omega, \Psi_1, \Psi_2, \ldots, \Psi_K) \) is the desired variable magnitude characteristics, and \( \Psi_k \), for \( k = 1, 2, \ldots, K \), are spectral parameters that determines corresponding magnitude specifications. These spectral parameters may represent...
cutoff frequency, center frequency of passband, transition bandwidth, passband width etc. The idea is to meet proposed specification and to change desired magnitude response by varying a single spectral parameter in desired predefined range. The first step in design procedure is a pre-design of set of constant filters. Each constant filter corresponds to a single (sampled) value of one spectral parameter. Therefore, every spectral parameter is uniformly sampled, giving $M_k$ values. The number of constant filters to be designed is $M_1 M_2 \cdots M_K$.

After the set of constant filters has been designed, the next step is to find coefficients of the multidimensional polynomial approximation, regarding the coefficients of the constant filters as the ideal values. In the case of variable FIR filter, this means that our objective is to find the optimal FIR filter
\[
H(z_{12}, \ldots, z_i) = \sum_{n=0}^{N-1} h_n(z_{12}, \ldots, z_i) z^{-n}.
\]

(2)

The coefficients of this filter are multidimensional polynomials of spectral parameters
\[
h_n(z_{12}, \ldots, z_i) = \sum_{l_1=0}^{l_{11}} \cdots \sum_{l_i=0}^{l_{11}} \sum_{l_K=0}^{l_{1K}} b(n, l_{11}, \ldots, l_i, \ldots, l_K) z_{12}^{l_1} \cdots z_i^{l_i} \cdots z_K^{l_K}.
\]

(3)

The optimal polynomial coefficients are found by minimizing the following squared approximation error given in Eq. (4).

The minimization of the error $E$ can be done using linear approach as follows [2]:

\begin{enumerate}
  \item **Step I**: Map the multidimensional index $(m_1, m_2, \ldots, m_k)$ to the 1-D index $m$ as
  \[
  m = (m_1 - 1) M_2 M_3 \cdots M_k + (m_2 - 1) M_3 \cdots M_k + \cdots + (m_k - 1) M_k + m_k,
  \]
  where
  \[
  m_i \in \{1, 2, \ldots, M_i\},
  \]
  \[
  i \in \{1, 2, \ldots, K\},
  \]
  \[
  M = M_1 M_2 \cdots M_K.
  \]

  **Step II**: Map the multidimensional index $(l_1, l_2, \ldots, l_K)$ to the 1-D index $l$ as
  \[
  l = (l_1 - 1) L_2 L_3 \cdots L_K + (l_2 - 1) L_3 \cdots L_K + \cdots + (l_K - 1) L_K + l_K,
  \]
  where
  \[
  l_i \in \{1, 2, \ldots, L_i\},
  \]
  \[
  i \in \{1, 2, \ldots, K\},
  \]
  \[
  L = (L_1 + 1)(L_2 + 1) \cdots (L_K + 1).
  \]

  **Step III**: Based on above index mappings, calculate
  \[
  E = \sum_{m_1=1}^{M_1} \sum_{m_2=1}^{M_2} \cdots \sum_{m_K=1}^{M_K} \sum_{l_1=0}^{l_{11}} \cdots \sum_{l_K=0}^{l_{1K}} \left[ B(n, l) \Phi_{m_1} - a(m) \right]^2.
  \]
  (4)

  In order to minimize $E$, we differentiate it with respect to each coefficient $B(n, l)$. Finally, the set of linear equations is obtained

  \[
  \Phi = \begin{bmatrix} \Phi_{t1} & \Phi_{t2} & \cdots & \Phi_{tL} \\ \Phi_{21} & \Phi_{22} & \cdots & \Phi_{2L} \\ \vdots & \vdots & \ddots & \vdots \\ \Phi_{M1} & \Phi_{M2} & \cdots & \Phi_{ML} \end{bmatrix}
  \]
  \[
  B(n) = \begin{bmatrix} B(n, 1) \\ B(n, 2) \\ \vdots \\ B(n, L) \end{bmatrix}
  \]
  \[
  \Phi^T a(n) = \begin{bmatrix} a(n, 1) \\ a(n, 2) \\ \vdots \\ a(n, M) \end{bmatrix}
  \]

  The coefficients obtained by solving Eq. (Error! Not a valid link.) are optimal in least-squares sense. It should be pointed out that the polynomial orders $L_k$ should be chosen with the tradeoff between design accuracy and computational complexity. It has been also shown in [3], that the weight functions can be selected such that the frequency response errors are almost uniformly distributed along all spectral parameters.

3. IMPLEMENTATION OF VARIABLE FILTERS

This section presents an effective implementation form for the variable FIR filter.

In order to derive the implementation structure it is desired to rearrange Eq. (2) as

\[
H(z_{12}, \ldots, z_i) = \sum_{l_1=0}^{l_{11}} \cdots \sum_{l_i=0}^{l_{1i}} \sum_{l_K=0}^{l_{1K}} \sum_{n=0}^{N-1} b(n, l_{11}, \ldots, l_i, \ldots, l_K) z_{12}^{l_1} \cdots z_i^{l_i} \cdots z_K^{l_K}.
\]

(17)

If it is set

\[
H_{l_{12} \cdots l_{1k}}(z) = \sum_{n=0}^{N-1} b(n, l_{11}, \ldots, l_i, \ldots, l_K) z^{-n},
\]

(18)

then the overall transfer function is written as

\[
H(z_{12}, \ldots, z_i) = \sum_{l_1=0}^{l_{11}} \cdots \sum_{l_i=0}^{l_{1i}} \sum_{l_K=0}^{l_{1K}} H_{l_{12} \cdots l_{1k}}(z) z_{12}^{l_1} \cdots z_i^{l_i} \cdots z_K^{l_K}.
\]

(19)

Here, $H_{l_{12} \cdots l_{1k}}(z)$ can be seen as a constant filter that corresponds to the weighting coefficient $z_{12}^{l_1} \cdots z_i^{l_i} \cdots z_K^{l_K}$. The weighting coefficients are variable part of the variable FIR filter. The overall structure contains $(L_1 + 1)(L_2 + 1) \cdots (L_K + 1)$ fixed filters of length $N$, and additional network of multipliers with corresponding weighting coefficients. The fixed FIR subfilters may be made symmetric or antisymmetric if the tuning ranges of spectral parameters are defined as $z_{12}^{l_1} \in [-z_{12}^{l_{1max}}, z_{12}^{l_{1max}}]$. Exploiting the coefficients symmetry or antisymmetry the presented structure has
(\(L_1+1\))(\(L_2+1\))(\(L_K+1\))\(N/2\) fixed coefficient multipliers. The overall structure contains \(L_1L_2...L_K\) variable multipliers. These variable multipliers in many applications can be made simple, as their wordlength can be several bits.

As it can be seen from Eq. (Error! Not a valid link.) the desired frequency characteristics can be obtained and tuned during operation by changing the corresponding spectral parameter \(\Psi\), while the coefficients of the fixed FIR subfilters are kept unchanged. The resolution of any spectral parameter is limited only by the precision of arithmetic used and not by the size of memory, as for example in coefficient memory approach. These characteristics make this structure a very attractive to be implemented using a VLSI circuit or a signal processor.

The special case of the above structure is the known Farrow structure [4]. In this case, the coefficients of the variable FIR filter are presented as 1-D polynomials of corresponding spectral parameter. The Farrow structure has been mainly used for implementation of variable fractional delay filters, see Fig. 2. However, possible applications of the Farrow structure are as follows: fractional delay filters, filters with variable magnitude response, beamforming filters etc. The fractional delay filter and its design are considered further in the design example section.

4. DESIGN EXAMPLE

This section illustrates through an example the effectiveness of the proposed design and implementation methods overviewed in this paper. The example has been adopted from [3]. The purpose of this example is illustration of the applications of digital variable filters. However, the efficient implementation and design of these filters will be topic of our further research.

It is desired to design a variable lowpass FIR filter with programmable fractional delay and changeable passband edge (variable magnitude characteristics). The desired lowpass filter is given as

\[H_f(\omega, \Psi, d) = M_f(\omega, \Psi)e^{j\theta_f(\omega, d)}\]  

(20)

with specifications

\[M_f(\omega, \Psi) = \begin{cases} 1 & 0 \leq \omega \leq \omega_p \\ \frac{\omega_p - \omega}{0.24\pi} & \omega_p \leq \omega \leq \omega_s \\ 0 & \omega_s \leq \omega \leq \pi \end{cases}\]  

(21)

\[\omega_p = 0.26\pi + \Psi, \ \omega_s = 0.50\pi + \Psi, \ \Psi \in [-0.16\pi, 0.16\pi]\]

(22)

Here, \(\omega_p\) and \(\omega_s\) are passband and stopband edges, respectively. The spectral parameter \(\Psi\) controls the passband and the stopband edges positions, while the transition bandwidth is fixed, and in this case, it is equal to 0.24\(\pi\). The fractional delay \(d\) and spectral parameter \(\Psi\) are defined in symmetric ranges, thus this assures symmetry and antisymmetry of fixed FIR subfilters. As it was mentioned earlier this property is desired in order to decrease the number of fixed coefficients multipliers.

The required filter can be designed using the presented procedure. The spectral parameters \(d\) and \(\Psi\) are first sampled and the set of discrete points is obtained, where \(M_1=17\) and \(M_2=11\). Then the variable filter having FIR subfilter length of

Fig. 1. General implementation form for variable FIR filter.

Fig. 2. Farrow structure as fractional delay filter.

Fig. 3. Variable magnitude response for \(d=0\).

Fig. 4. Variable fractional delay for response for \(\Psi=0\).
The filter order should be chosen with the tradeoff between design accuracy and computational complexity. The performances of the designed filter are shown in the following two figures. Figure 3 shows the actual variable magnitude response for $d=0$. Figure 4 shows the passband fractional phase delay for $\Psi=0$.

The designed tunable variable filter can be implemented using 2-D polynomial version of the structure shown in Fig. 2. For $\Psi=0$ or $d=0$ this is reduced to the known Farrow structure which was earlier used. The number of fixed FIR subfilters is equal $(L_1+1)(L_2+1)=25$. The FIR subfilter length is $N=30$. It should be pointed out that these FIR subfilters are symmetric or anti-symmetric. The overall number of fixed coefficients is equal $(L_1+1)(L_2+1)N/2=375$. In variable part of the structure, there exist $L_1L_2=16$ multipliers.

5. CONCLUSIONS

The effective design technique for variable FIR filters has been overviewed. The key idea is to approximate the coefficients of the variable FIR filter with the multidimensional polynomial functions of spectral parameters. Then, the linear least square optimization method is used to obtain optimal coefficients. The implementation structure can be extension of the known Farrow structure to multidimensional polynomial case. The main advantage of this type of structure is that it has only one changeable parameter associated with each spectral parameter, and set of fixed symmetric or antisymmetric FIR subfilters. These characteristics of the implementation structure make it a very attractive structure to be implemented using a VLSI circuit or a signal processor.

REFERENCES:


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Ovaj rad takodje ispituje mogucnost realizacije i prikladne strukture za realizaciju za ovaj tip filutra. Za razmotrenu strukturu je karakteristично to da je njena osoba u frekventnom domenu može podestiti promenom samo jednog parametra koji je povezan sa njom, dok se koeficijenti FIR podfiltara ne menjaju.

DIZAJN I RELAZIACIJA PROMENLJIVIH (SA MOGUCNOSCUC PROGRAMIRANJA) FIR FILTARA, D. Babic