FIBER BANDWIDTH AROUND ZERO DISPERSION WAVELENGTH

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I INTRODUCTION

In calculation of fiber pulse response in transmission through a monomode fiber, when a coherent optoelectronics communication system is considered, one can make use of the Gaussian pulse transmission as described in [1]. In that case it is assumed that the complex input impulse is of the form

\[ e_0(t) = E_0 \exp \left( -\frac{t^2}{2\sigma_0^2} + j\Omega_0 t \right) \quad (1) \]

where \( \Omega_0 \) is the optical frequency and \( \sigma_0 \) is the standard deviation of the input pulse envelope. For the phase coefficient of a fiber in the form,

\[ \beta(\Omega) = \beta_0 + \beta'_0 (\Omega - \Omega_0) + \frac{\beta''_0 (\Omega - \Omega_0)^2}{2} \quad (2) \]

where \( \beta_0 = \beta(\Omega_0) \), \( \beta'_0 = \beta'(\Omega_0) \), \( \beta''_0 = \beta''(\Omega_0) \), a closed form solution was found for the complex output signal in the form of another Gaussian pulse:

\[ e_L(t) = \sqrt{\frac{\Omega_0}{\sigma_L}} E_0 \exp \left[ j \left( \Omega_0 t - \beta_0 L \right) - \frac{(t - \beta'_0 L)^2}{2\beta'_0 L_L} - \Psi \right] \times \exp \left( -\frac{(t - \beta'_0 L)^2}{2\sigma_L^2} \right) \quad (3) \]

where

\[ \Psi = \frac{1}{2} \arctan \frac{\beta''_0 L_L}{\sigma_0^2}, \quad \sigma_L = \sigma_0 \left[ 1 + \left( \frac{\beta''_0 L_L}{\sigma_0^2} \right)^2 \right]^{1/2} \]

\[ L_L = \left[ L + \left( \frac{\sigma_0^2}{\beta''_0 L} \right)^2 \right]^{1/2} \quad (4) \]

When the total dispersion \( d_1(\lambda) \) is zero, \( \beta'_0 \) is zero, so that we have \( \sigma_L = \sigma_0, \Psi = 0 \) and we have to find the limiting value of \( \beta''_0 L_L \) when \( \beta'_0 \to 0 \), i.e. Thus, if Eq.(2) is assumed, the closed form solution given by Eq.(3) would be

\[ e_L(t) = E_0 \exp \left[ j \Omega_0 t - \beta_0 L \right] \cdot \exp \left( -\frac{(t - \beta'_0 L)^2}{2\sigma_0^2} \right) \quad (5) \]

which would mean that there would be no distortion of the pulse and only delay would be present. However, in this case we have to introduce, at least, another term in Eq.(2) and look for the output pulse under such condition.

Following the same procedure in developing integral representing the output pulse, we come to the integral of the form (see Eq.(4.11) in [1]):

\[ e_L(t) = \frac{E_0 \sigma_0}{\sqrt{2\pi}} \exp(j\Omega_0 t) \times \int_{-\infty}^{\infty} \exp \left( -z^2 \sigma_0^2 / 2 \right) \exp(-j\beta L + jwt) dz \quad (6) \]

Where \( z = \Omega - \Omega_0 \), and \( \beta \) is as given by Eq.(2) with an additional term \( \beta''_0 (\Omega - \Omega_0)^3 L/3! \). As the closed form solution to the integral (6) is then not known, we solved numerically some special case of the step-index fibre close to the specification of standard monomode fibre (G652).

II PHASE COEFFICIENT OF THE STANDARD FIBRE

Based on data given in [3], we calculated the core and cladding refractive indices \( n_c(\lambda), n_a(\lambda) \), and the group refractive indices \( N_c(\lambda), N_a(\lambda) \). The fiber radius \( a \) is obtained from selected cutoff wavelength \( \lambda_c = 1.28 \mu m \):

\[ a = \frac{2.405 \lambda_c}{2\pi \sqrt{n_c^2(\lambda_c) - n_a^2(\lambda_c)}} = 3.6368 \mu m \quad (7) \]

The next step is calculation of the normalized frequency fiber parameter as the function of the wavelength:

\[ V(\lambda) = k_0(\lambda) a \sqrt{\frac{n_c^2(\lambda) - n_a^2(\lambda)}} \]

Then we solved the characteristic equation for the LP_{01} wave

\[ \frac{J_0(U)}{U J_1(U)} = \frac{K_0(W)}{W K_1(W)} \quad (8) \]

where \( W^2 = V^2 - U^2 \). The phase coefficient is calculated from

\[ \beta(\lambda) = \sqrt{\left( k_0 n_c \right)^2 - \frac{U^2}{a^2}} \quad (9) \]

Since we need expansion in optical frequency \( \Omega \), and the phase coefficient is, as calculated by (8), a function of \( \lambda \), we used polynomial expansion with \( \beta(\Omega) \) in terms of \( \Omega = 2\pi c / \lambda \), where \( c \) is the free space light velocity. In this way we obtained a third order polynomial for \( \beta(\Omega) \) for seven values of \( \Omega \), corresponding to the wavelengths in the range...
1.2 to 1.55 µm. The \( \beta(\lambda) \) is calculated in \( \mu m^{-1} \), and \( \Omega \) is in \( 10^{14} \)Hz units:

\[
\beta(\Omega) = -0.1261336 + 0.49744789\Omega - 5.62062415 \cdot 10^{-4} \Omega^2 + 1.33024354 \cdot 10^{-5} \Omega^3
\]

(10)

A word of caution is needed here. When we tried to find similar expression to (10), but for the data in a narrower wavelength range, somewhat different expression is obtained. As our main aim was to see the effect of the cubic term in (10), we elaborated this question of ultimate accuracy of numerical calculation in an accompanying paper [2].

Conversion of \( \beta(\Omega) \) in the form given by Eq.(2), require selection of the center operating frequency \( \Omega_0 \) in order to obtain phase coefficient expansion around \( \Omega_0 \). A simple way to do this was to compare the two expansions

\[
\beta(\Omega) = a_0 + a_1 \Omega + a_2 \Omega^2 + a_3 \Omega^3
\]

(11)

\[
\beta(\omega) = b_0 + b_1 \omega + b_2 \omega^2 + b_3 \omega^3
\]

(12)

where \( \omega = \Omega - \Omega_0 \). The relationship between \( a_i \) and \( b_i \) parameters is found from the matrix equation

\[
\begin{pmatrix}
0 & 1 & \Omega_0 & \Omega_0^2
0 & 1 & 2\Omega_0 & 3\Omega_0^2
0 & 0 & 1 & 3\Omega_0
0 & 0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
a_0 \\
a_1 \\
a_2 \\
a_3 \\
\end{pmatrix} =
\begin{pmatrix}
b_0 \\
b_1 \\
b_2 \\
b_3 \\
\end{pmatrix}
\]

(13)

If we want to find \( \Omega_0 \) where the dispersion is zero, then we have to solve the equation

\[
b_2 = a_0 + 3a_3 \Omega_0 = 0
\]

(14)

which, for the coefficients obtained from Eq.(10), gives \( \Omega_{0(d=0)} = 14.084198 \) \( [10^{14} \text{Hz}] \), or \( \lambda_{0(d=0)} = 1.33835 \mu m \).

Table 1 shows variation of \( \beta_0^\prime, \beta_0^\prime\prime \) and the total dispersion \( d_i \) with wavelength.

<table>
<thead>
<tr>
<th>( \lambda_0 ) (( \mu m ))</th>
<th>( \beta_0^\prime(10^{-28} \frac{\mu m^2}{m}) )</th>
<th>( \beta_0^\prime\prime(10^{-42} \frac{\mu m^4}{m^3}) )</th>
<th>( d_i(\frac{ps}{nm km}) )</th>
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<td>1.25</td>
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</table>

**Table 1:** Approximate phase coefficient expansion parameters

### III Calculation of Pulse Shape without and with the Third Order Term

a) With the second order term only

As mentioned earlier, the inclusion of the second order term in Eq.(2) leads to the closed form solution. The output pulse is the Gaussian shape and the relationship between the bit rate and the pulse standard deviation can be obtained on assumption that the eye opening is reduced by specified dB penalty. The worst case closure of the eye can be obtained for a pulse combination 1-0-1, as compared to 0-1-0 combination. By assuming 1 dB allowance for the eye-closure, the relationship between the bit rate \( f_{br} = 1/T \), and the standard deviation is obtained from the expression

\[
20 \log \left[ 1 - 2 \exp \left( -\frac{T^2}{2\sigma_L^2} \right) \right] = -1 \text{dB} \tag{15}
\]

or

\[
f_{br} = \frac{0.4144}{\sigma_L} \tag{16}
\]

The amplitude decay due to the pulse widening is from Eq. (3)

\[
a_{\sigma} = 10 \log \left( \frac{\sigma_0}{\sigma_L} \right) \tag{17}
\]

and this loss has to be included in the calculation of the overall loss in addition to the attenuation due to power decay caused by the fiber attenuation coefficient.

From Eq.(16) and Eq.(3) we find the dependance \( L(f_{br}) : \)

\[
L(f_{br}) = \frac{\sigma_0}{\beta_0} \left( \frac{0.4144}{f_{br}} \right)^2 \tag{18}
\]

From Eq.18 it is seen that the length for the specified bit rate depends not only on the fiber dispersion (proportional to \( \beta_0^\prime \)), but also on \( \sigma_0 \), the initial pulse standard deviation.
Fig.1. Bit rate vs. distance for pulses 3, 4, 5, 6, 7 ps for a wavelength where the dispersion is 1 ps/(nm km)

b) With the second and the third term

In this case the cubic dependence of the phase coefficient \( \beta(\omega) \) are taken into account in calculating pulse shapes.

\[
F_1(t) := \int_{-1}^{1} e^{-5 \cdot 10^{4} x^2 + j x t - j \cdot 1.3124703 \cdot 10^4 L x^3 - j \cdot 1.0376159 \cdot 10^3 L x^2} dx
\]

\[
F_2(t) := \int_{-1}^{1} e^{-5 \cdot 10^{4} x^2 + j x t - j \cdot 1.0376159 \cdot 10^3 L x^2} dx
\]

Finally, we calculated eye-diagrams for several cases as shown in Fig.4 and 5.
Abstract:

In this paper, the effect of the third order term of the singlemode fiber phase coefficient on the shape of the output pulse is shown. As input, a Gaussian shaped pulse is used. Also, the dependance of bitrate vs. distance is obtained, for different pulse widths, when the eye-closure penalty was fixed. By numerical calculation, it is shown that the effect of the cubic term in phase coefficient is not important for the fibers with dispersion greater than 1 ps/nm-km and for pulses longer than 1 ps.

IV CONCLUSION

By numerical calculation of the output pulse shapes it is proved that the theory developed in [1] can be used even for smaller dispersion of 1 ps/(nm km), and the cubic term in their phase coefficient can be neglected. Only for very short pulses (less than 1 ps) the effect of the cubic term is shown but even then if the dispersion is of the order 1 ps/(nm km).

LITERATURE

