ANALYSIS OF CASCADE-CONNECTED TRANSMISSION LINES WITH INCREASED WIDTHS BY ETS METHOD

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Abstract: Equivalent Thevenin Source (ETS) method is now used for analysis of two-dimensional circuit constructed as cascade-connected uniform transmission lines with different lengths and increased widths. New relations for multi-port network driven by current and voltage sources are given and presented procedure is verified on one example of microstrip stepped-impedance lowpass filter.

I. INTRODUCTION

Different concepts for modelling and analysis of a large class of two-dimensional circuit structures are given in the papers [1-8]. The papers [1-2] describe two different methods for analysis two-dimensional transmission line equivalent circuit. In both papers the line is characterized in term of its transmission matrices A, B, C and D to discuss its properties. In the paper [1] ETS method is given and analysis is based on decomposition that two-dimensional circuit into cascade-connected ladder subnetworks with same number of input and output ports. This method is much efficient than the method of direct multiplication of the individual chain matrices [2].

The ETS method described in the paper [1] is used for analysing both a single microstrip line [3] and cascade-connected uniform lines with different lengths and reduced widths [4]. Two-dimensional circuits are represented as cascade-connected networks with equal or reduced number of ports.

Described ETS method can be also applied to cascade connection of uniform lines with different lengths and increased widths. Two-dimensional circuits are represented as cascade-connected networks with different increased number of ports. The proposed procedure is verified on one example of a microstrip stepped-impedance lowpass.

II. MICROSTRIP DISCONTINUITY TYPES

Various types of discontinuities that occur in the conductor of planar transmission lines, such as microstrips, are shown in Fig.1. The junction of two lines having different widths forms step in width discontinuity. The input port is at the narrower line and the output port is at the wider line.

Fig.1. Typical microstrip discontinuities.

III. MULTI-PORT NETWORK DRIVEN BY CURRENT AND VOLTAGE SOURCES

The ladder network depicted in Fig.2 may be uniquely described by a set of equations relating port voltages and currents

\[ U_1 = A U_2 + B I_2, \]  
\[ I_1 = C U_2 + D I_2, \]

where A, B, C and D are transmission matrices of the network given in the paper [1]. The network is driven by \(k1+k2\) current and \(m\) voltage real sources and has \(2L_1\) ports, \(L_1 = k1 + m + k2\), as shown in Fig. 2.

Fig.2. Multi-port network driven by current and voltage sources.

The input voltage and current's vectors of the network can be written according to source disposition as follows

\[ U_1 = [U_{k1c} | U_{m,v} | U_{k2c}] = [U_{11} U_{12} \ldots U_{111}]^T \]

and

\[ I_1 = [I_{k1c} | I_{m,v} | I_{k2c}] = [I_{11} I_{12} \ldots I_{111}]^T. \]

The first sign of subscripts in the vectors indicates the number of the sources and the second one indicates the type of source (current or voltage source).

Also, the output voltage and current's vectors of the network can be written as follows

\[ U_2 = [U_{2k1} | U_{2m} | U_{2k2}] = [U_{21} U_{22} \ldots U_{211}]^T \]

and

\[ I_2 = [I_{2k1} | I_{2m} | I_{2k2}] = [I_{21} I_{22} \ldots I_{211}]^T. \]
Previously equation system can be written now as follows

\[
\begin{align*}
[U_{k1c}] & = \begin{bmatrix} A_{k11} & A_{k1m} & A_{k12} \\ A_{m11} & A_{m1m} & A_{m12} \\ A_{k21} & A_{k2m} & A_{k22} \end{bmatrix} \begin{bmatrix} U_{2,k1} \\ U_{2,m} \\ U_{2,k2} \end{bmatrix} + \\
[U_{k2c}] & = \begin{bmatrix} B_{k11} & B_{k1m} & B_{k12} \\ B_{m11} & B_{m1m} & B_{m12} \\ B_{k21} & B_{k2m} & B_{k22} \end{bmatrix} \begin{bmatrix} I_{2,k1} \\ I_{2,m} \\ I_{2,k2} \end{bmatrix}
\end{align*}
\]

(7)

\[
\begin{align*}
[I_{k1c}] & = \begin{bmatrix} C_{k11} & C_{k1m} & C_{k12} \\ C_{m11} & C_{m1m} & C_{m12} \\ C_{k21} & C_{k2m} & C_{k22} \end{bmatrix} \begin{bmatrix} U_{2,k1} \\ U_{2,m} \\ U_{2,k2} \end{bmatrix} + \\
[I_{k2c}] & = \begin{bmatrix} D_{k11} & D_{k1m} & D_{k12} \\ D_{m11} & D_{m1m} & D_{m12} \\ D_{k21} & D_{k2m} & D_{k22} \end{bmatrix} \begin{bmatrix} I_{2,k1} \\ I_{2,m} \\ I_{2,k2} \end{bmatrix}
\end{align*}
\]

(8)

In order to form real input vector with known voltages, \( U_{m,v} \), and currents, \( I_{k1c} \) and \( I_{k2c} \), respectively, the permutation of rows in the existing transmission matrices must be done. After row permutation is done, the system becomes

\[
\begin{align*}
[U_{k1c}] & = \begin{bmatrix} I_{s,k1} - Y_{s,k1} U_{k1c} \\ U_{s,m} - Z_{s,m} m,v \\ I_{s,k2} - Y_{s,k2} U_{k2c} \end{bmatrix} = \\
[U_{k2c}] & = \begin{bmatrix} C_{k11} & C_{k1m} & C_{k12} \\ A_{m11} & A_{m1m} & A_{m12} \\ C_{k21} & C_{k2m} & C_{k22} \end{bmatrix} \begin{bmatrix} U_{2,k1} \\ U_{2,m} \\ U_{2,k2} \end{bmatrix} + \\
& = \begin{bmatrix} D_{k11} & D_{k1m} & D_{k12} \\ B_{m11} & B_{m1m} & B_{m12} \\ D_{k21} & D_{k2m} & D_{k22} \end{bmatrix} \begin{bmatrix} I_{2,k1} \\ I_{2,m} \\ I_{2,k2} \end{bmatrix}
\end{align*}
\]

(9)

It can be concluded that output voltage and current's vectors don't have changes after row permutations.

New formed matrices with permuted rows can be further assign as \( A_E \), \( B_E \), \( C_E \) and \( D_E \). After taking into the system new signs for matrices it can be written

\[
\begin{align*}
[I_{s,k1}] & = \begin{bmatrix} Y_{s,k1} & 0 & 0 \\ 0 & Z_{s,m} & 0 \\ 0 & 0 & Y_{s,k2} \end{bmatrix} \begin{bmatrix} U_{k1c} \\ U_{k2c} \end{bmatrix} = \begin{bmatrix} A_E & U_2 + B_E \end{bmatrix},
\end{align*}
\]

(10)

\[
\begin{align*}
[U_{k1c}] & = \begin{bmatrix} A_{k11} & A_{k1m} & A_{k12} \\ C_{m11} & C_{m1m} & C_{m12} \\ A_{k21} & A_{k2m} & A_{k22} \end{bmatrix} \begin{bmatrix} U_{2,k1} \\ U_{2,m} \\ U_{2,k2} \end{bmatrix} + \\
[U_{k2c}] & = \begin{bmatrix} B_{k11} & B_{k1m} & B_{k12} \\ D_{m11} & D_{m1m} & D_{m12} \\ B_{k21} & B_{k2m} & B_{k22} \end{bmatrix} \begin{bmatrix} I_{2,k1} \\ I_{2,m} \\ I_{2,k2} \end{bmatrix}
\end{align*}
\]

(11)

\[
\begin{align*}
[I_{s,k1}] & = \begin{bmatrix} Y_{s,k1} & 0 & 0 \\ 0 & Z_{s,m} & 0 \\ 0 & 0 & Y_{s,k2} \end{bmatrix} \begin{bmatrix} U_{k1c} \\ U_{k2c} \end{bmatrix} = \begin{bmatrix} C_E & U_2 + D_E \end{bmatrix},
\end{align*}
\]

(12)

Substituting the relation (12) into (11), the source vector is

\[
\begin{align*}
[U_{s,m}] & = \begin{bmatrix} 0 & Z_{s,m} & 0 \\ 0 & 0 & Y_{s,k2} \end{bmatrix} \begin{bmatrix} U_{2} + B_E \end{bmatrix} + \\
[I_{s,k2}] & = \begin{bmatrix} A_E & + \begin{bmatrix} Y_{s,k1} & 0 & 0 \\ 0 & Z_{s,m} & 0 \\ 0 & 0 & Y_{s,k2} \end{bmatrix} \end{bmatrix} \begin{bmatrix} U_2 + \\ U_{2,m} \\ U_{2,k2} \end{bmatrix}
\end{align*}
\]

(13)

where first sign of subscripts indicates the source currents or voltages and the second one indicates the number of those sources.
The voltage vector of ETS for open-ended network is
\[
U_{2T} = \begin{bmatrix} \mathbf{A}_E + \left[ \begin{array}{cc} Y_{s,k1} & 0 \\ 0 & Z_{s,m} \\ 0 & 0 \end{array} \right] C_E \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ Y_{s,k1} \\ U_{s,m} \\ Y_{s,k2} \end{bmatrix} 
\] (14)
and the impedance matrix of ETS for annulled current and voltage sources is
\[
Z_{2T} = \begin{bmatrix} \mathbf{A}_E + \left[ \begin{array}{cc} Y_{s,k1} & 0 \\ 0 & Z_{s,m} \\ 0 & 0 \end{array} \right] C_E \end{bmatrix}^{-1} 
\] (15)

The relations (14) and (15) are equivalent to the recurrent relations (16) and (17) given in the paper [1].

III. ETS Voltage and Impedance Calculation

Cascade-connected planar transmission lines of different increased widths and different lengths can be analysed as cascade-connected networks with different number of input and output ports. Junction of two networks with increased number of input ports is shown in Fig. 3. That cascade connection corresponds to the discontinuity (i) given in Fig. 1. The first and the other networks including the k\textsuperscript{th} network as the last one have 2L ports. The k + 1\textsuperscript{st} network and all networks till the end have 2L_1 ports, where L_1 > L. The voltage and impedance’s superscript indicates the number of network in cascade connection.

First k cascade-connected networks with the same number of input and output ports can be analysed by ETS method [1]. Voltages and impedances of the k\textsuperscript{th} network can be recovered from the recurrent relations
\[
U_{2T}^{k+1} = [\mathbf{A}_E + Z_{2T}^{-1}C_k]^{-1}U_{2T}^{k-1},
\] (16)
\[
Z_{2T}^{k+1} = [\mathbf{A}_E + Z_{2T}^{-1}C_k]^{-1}[B_k + Z_{2T}^{-1}D_k].
\] (17)

It can be shown that these recurrent relations can be used also for the next cascade-connected network with different increased number of ports.

The impedance matrix obtained by the equation (17) is full matrix
\[
Z_{2T}^k = \begin{bmatrix} Z_{21}^k & \ldots & Z_{2L_1}^k \\ \vdots & \ddots & \vdots \\ Z_{21}^k & \ldots & Z_{2L_1}^k \end{bmatrix}.
\] (18)

According to the network connection given in Fig. 3 the source impedance matrix for the next k + 1\textsuperscript{st} network is impedance matrix increased as
\[
Z_s = \begin{bmatrix} 0 & 0 & 0 & 1 \_k1 \\ \_k-1 & \ldots & \_k-1 & 0 \\ \_k-1 & \ldots & \_k-1 & 0 \\ 0 & 0 & 0 & 1 \_m \_m \\ 1 & \_k1 & \_m & \_k2 \end{bmatrix}
\] (19)
where 0 is zero matrix.

The voltage vector of ETS at the k\textsuperscript{th} open-ended network is full vector
\[
U_{2T}^k = \begin{bmatrix} u_{2T,1}^k \ldots u_{2T,L_1}^k \end{bmatrix}
\] (20)
and it can be increased in the form
\[
U_s = \begin{bmatrix} 0 \ldots 0 | u_{2T,1}^k \ldots u_{2T,L_1}^k | 0 \ldots 0 \end{bmatrix} = \begin{bmatrix} 0 | U_{2T}^k | 0 \end{bmatrix}
\] (21)
which represents the source vector of the next k + 1\textsuperscript{st} network.

The voltage vector and the impedance matrix of ETS for the k + 1\textsuperscript{st} open-ended network are
\[
U_{2T}^{k+1} = [\mathbf{A}_E + Z_s C_s]^{-1}U_s
\] (22)
\[
Z_{2T}^{k+1} = [\mathbf{A}_E + Z_s C_s]^{-1}[B_s + Z_s D_s].
\] (23)

These relations can be obtained from the relations (14) and (15) for Y_{s,k1} = 0, Y_{s,k2} = 0, I_{s,k1} = 0 and I_{s,k2} = 0.

The last two recurrent relations are equivalent to the recurrent relations (16) and (17).

The solving procedure for cascade-connected networks with increased number of ports is as follows:

1. The relations (16) and (17) are used to obtain \( U_{2T}^k \) and \( Z_{2T}^k \), i.e. ETS voltages and impedances for the first k cascade-connected networks. These vector and matrix are applied to the input ports of the next k + 1\textsuperscript{st} cascade-connected network.
2. The matrices \( \mathbf{A}_E \), \( \mathbf{B}_E \), \( \mathbf{C}_E \) and \( \mathbf{D}_E \) are formed for the k + 1\textsuperscript{st} network with 2L_1 ports.
3. At the junction between the k\textsuperscript{th} and k + 1\textsuperscript{st} networks, because of the increased number of input ports, it is necessary to increase the vector \( U_{2T}^k \) and the matrix \( Z_{2T}^k \) as shown in relations (21) and (19). The voltage vector \( U_{2T}^{k+1} \) and impedance matrix \( Z_{2T}^{k+1} \) are calculated from relations (22) and (23).
4. For the further calculation, k + 2, k + 3, ... \( K \), the relations (16) and (17) can be used for solving the rest of the networks in cascade connection.

The suggested procedure is incorporated in program \textit{FAMIL} (Frequency Analaysis of Microwave Lines) created in \textit{MATLAB}.

V. Example

Consider a microstrip stepped-impedance 7\textsuperscript{th} order Chebyshev lowpass filter with a cutoff frequency of 900 MHz and 500 ohm terminations given in the chapter 7.2
in the reference [10]. A layout diagram is shown in Fig.4. The nominal substrate dielectric constant is $\varepsilon_r = 6.0$, the board thickness is $h = 635 \mu m$ and the strip thickness is $t = 0$. The total length of the filter with leaders exceeds 104.14 mm.

The stepped-impedance lowpass is a cascade of alternating high and low impedance transmission lines. The high impedance lines act as series inductors and the low impedance lines act as shunt capacitors. The high impedance narrow lines are $465.18576 \mu m$ wide (line2, line4, line6 and line8) and the low impedance wide lines are $4811.522 \mu m$ wide (line3, line5 and line7). The intermediate width lines at the ends ($w_1 = w_9 = 932.59402 \mu m$) are the $500 \Omega$ leader lines. If the filter is terminated in $500 \Omega$, the lengths of these leaders affect only the dissipation loss and phase length of the filter.

This microstrip lowpass is observed as cascade-connected uniform transmission lines with different lengths and increased or reduced widths. Their lengths are $d_1 = d_9 = 2540 \mu m$, $d_2 = d_8 = 1811.49016 \mu m$, $d_3 = d_7 = 7834.4982 \mu m$, $d_4 = d_6 = 32042.0238 \mu m$ and $d_5 = 9771.33174 \mu m$.

The results obtained by program FAMIL are compared with ones obtained by program GENESYS [11] and shown in Fig.5. The attenuation characteristics are in good agreement at lower frequencies. For higher frequencies the results differ slightly.

**VI. CONCLUSION**

The ETS method for frequency analysis of microwave lines given in the papers [1,3 and 4] here is extended for analysis of cascade-connected uniform transmission lines with different lengths and increased widths. The relations needed for analysis of such circuits are derived.

Described procedure is used for analysis of one Chebyshev lowpass filter realised in microstrip lines technique [10]. The analysis results from program FAMIL showed good agreement with the ones obtained by program GENESYS [11]. A presented procedure involves various types of discontinuities shown in Fig.1. Such discontinuities are analyzed as two-dimensional circuits and because of that assume a longer computing time than that classical procedure given in program GENESYS.

Although the present ETS method applied only to tapered lines, it can be further generalized to analyze any nonuniform transmission lines.

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**REFERENCES**


