DISTANCE ANALYSIS FOR E²PR4 TWO-TRACK TWO-HEAD MAGNETIC RECORDING CHANNEL

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Abstract – Distance analysis of uncoded E²PR4 two-track two-head magnetic recording channel was considered. Two-track two-head system with E²PR4 partial response in-track intersymbol interference model is used, assuming linear symmetrical intertrack interference. Generalised search criterion for finding input error sequences that negatively effect on two-track two-head square Euclidean distance is given.

I. INTRODUCTION

Growing demand for massive amounts of data storage caused that high capacity magnetic storage devices are increasingly used in consumer electronics area. Over the past two years the annual growth rate in storage devices areal density is a whopping 100% [1]. Initial step in areal density increase is the suitable physical organization of recorded data on storage media. In traditional magnetic recording systems the data are recorded in tracks as a sequence of small magnetic domains.

Most prior research in disk drive systems has been focused on increasing areal density by reducing the length of magnetic domains along tracks (increasing linear density). The magnetic domains become smaller and thus thermally unstable, which means that lower energy of an external magnetic field is sufficient to demagnetise them. This effect is known as a super paramagnetic effect [2]. Linear recording density is also limited by read head ability to accurately detect and decode recorded data in the presence of the high intersymbol interference (ISI).

The limitations of linear densities [3] encourage development of another strategy for increasing the areal density. One of them uses approach to reduce the track width and track pitch (increasing track, or radial, density), and it is a very promising way for future days. In this approach the one traditional magnetic recording track is divided into M new tracks with decreased width, as shown in Fig. 1.

![Fig. 1. Increase in radial density](image)

In order to increase radial density, multiple-head arrays have been developed enabling reading and writing data simultaneously on multiple tracks [4]. Such heads can potentially provide both high density and high speed [5], [6], but they suffer from intertrack interference (ITI). This ITI is a result of a signal induced in the heads due to the superposition of magnetic transitions in neighboring tracks.

Soljanin and Georgiades have shown that multiple-head systems can better combat intertrack interference [7], which seriously degrades the error-rate performance of single-head detector, in high-density magnetic recording systems. In addition, for writing and reading on multiple tracks simultaneously the required redundancy for timing and gain control can be reduced. Furthermore, multiple-head systems have shown to be more robust to the head misalignment errors [8].

This paper considers the square Euclidean distance (s.Ed.) analysis of E²PR4 two-track two-head (E²PR4 TTTH) magnetic recording channel, in presence of intertrack interference (ITI). TTTH channel model is presented in Section II. Square Euclidean distance analysis and critical error sequence search results are presented in Section III and Section IV. Finally, concluding remarks on the paper are presented in Section V.

II. TWO-TRACK TWO-HEAD CHANNEL MODEL

The E²PR4 TTTH ITI-based magnetic recording channel, with additive linear and symmetrical ITI modelled with the following matrix

\[ A = \begin{bmatrix} 1 & \varepsilon \\ \varepsilon & 1 \end{bmatrix} \]  (1)

where \( \varepsilon \in [0, 1] \) represents ITI level between tracks [9], [10], was assumed, as shown in Fig. 2.

![Fig. 2. Two-track two-head ITI-based channel model](image)

The individual in-track channels are equalized to the E²PR4 R4 partial-response (PR) model with transfer polynomial of the form \( P(D)=(1-D)(1+D)^2 \) [11], modelling in-track intersymbol interference (ISI).

Denote the in-track channel input sequence of length \( L<\infty \), in the track \( k \), \( k \in \{1, 2\} \), with \( s_k = \{s_{ki} \in (-1, +1)\}^L \), and with

\[ X_k(D) = \sum_{i=-\infty}^{\infty} x_{ki} D^i \]  (2)

corresponding \( D \)-transform. Consider the set of all admissible input error sequences in the track \( k \) [12]

\[ e_{ck}(D) = [X_k(D) - X'_k(D)]/2, \]  (3)

where \( X_k(D) \) and \( X'_k(D) \) represent two possible data input sequences, respectively. The squared Euclidean distance of the in-track input channel error sequence \( e_{ck} \), defined as

\[ d^2(e_{ck}) = \|e_{ck}(D)\|^2 = \|e_{ck}(D)P(D)\|^2 = \sum_{i=-\infty}^{\infty} e_{ki}^2, \]  (4)
have significant influence on the maximum likelihood one-head detector performance. It is well known that at moderate- to-high level of the signal-to-noise (SNR) ratio and in the presence of AWGN noise, the error probability performance of one-head detector, in the track $k$, is dominantly governed by

$$d_{\text{min}}^2 = \min_{e_{1,2}}(d^2(e_{1,2})), \quad (5)$$

i.e. the minimum squared Euclidean distance (m.s.d.), of the track $k$.

### III. TWO-TRACK TWO-HEAD DISTANCE ANALYSIS

Denoting the corresponding $D$-transform of data output sequences from TTTH ITI-based channel with

$$F_i(D) = 1 + e \cdot [P(D) + 1] \cdot (P(D))$$

the TTTH output error sequences are

$$\begin{align*}
    e_{1}(D) &= 1 + e \cdot e_{1}(D) \\
    e_{2}(D) &= 1 + e \cdot e_{2}(D)
\end{align*} \quad (7)$$

where $e_{1}(D)$ is output error sequence in the track $k$, $k \in \{1,2\}$, defined as

$$e_k(D) = \frac{[f_k(D) - f_k'(D)]}{2}, \quad (8)$$

and $e_{1}(D)$ represents filtered in-track input error sequence in the track $k$, defined as

$$e_{d_k}(D) = P(D)e_{1,2}(D) = \sum_{i=-\infty}^{\infty} e_{1,2}(D)D^i. \quad (9)$$

The s.E.d. of input error sequence pair $(e_{1}, e_{2})$ in two-track interfering channel with two-head detector is

$$d_{2D}^2(e_{1}, e_{2}) = \| e_{1}(D) \|^2 + \| e_{2}(D) \|^2 = (1 + e^2) \| d_1^2(e_{1}) + d_2^2(e_{2}) \|^2 + 4e \sum_{i=-\infty}^{\infty} e_{1,2}(D)D^i, \quad (10)$$

where $d_{1}^2(e_{1})$ and $d_{2}^2(e_{2})$ are s.E.d. produced by error sequence $e_{1}$ and $e_{2}$ in the corresponding track $k$. Because the $d_{k}^2(e_{1,2}) > 0$ (4), for all possible in-track input error sequences the TTTH s.E.d. most interesting cases are the following.

**Case a** When the combination of two-track input error sequences $(e_{1}, e_{2})$ are present at the input of TTTH system, such that

$$\begin{align*}
    e_{1,2} &= 0 \wedge e_{1,2} \neq 0 \\
    e_{1,2} &\neq 0 \wedge e_{1,2} = 0; \quad \forall i,
\end{align*} \quad (11)$$

which means that error sequence is present in one track only; assume $(e_{1}, 0)$ case. The s.E.d. becomes

$$d_{2D}^2 = (1 + e^2) \cdot d_1^2(e_{1}), \quad (12)$$

as is depicted in Fig. 3. The TTTH s.E.d. growth can be noticed, when the ITI level increasing is present.

Assumption of first track input error sequence presence is not limiting factor for generalization of s.E.d. analysis. The analogue result can be obtained when the second track is in focus of the observation.

**Case b** The second interesting case is the presence of input error sequence pair $(e_{1}, e_{2})$, such that

$$e_{1,2} \cdot e_{1,2} < 0 \quad \wedge \quad |e_{1,2}| = |e_{1,2}|; \quad \forall i, \quad (13)$$

which means that error pair $(e_{1}, e_{2})$ contains the opposite sign error pattern, $e_{2} = -e_{1}$, and the s.E.d. becomes

$$d_{2D}^2 = 2 \cdot (1 - e^2) \cdot d_1^2(e_{1}), \quad (14)$$

In this case the TTTH s.E.d. monotonically decreases with ITI level, as shown in Fig. 3.

![Fig. 3. s.E.d. of E^3PR4 TTTH ITI-based channel](image)

Note that for $\varepsilon = \varepsilon_o = 0.268$ s.E.d. in Case a) and b) are

$$d_{2D}^2 = d_{2D}^2 b, \quad (13)$$

so the performance of the TTTH system is controlled by two different segments of s.E.d. versus ITI curve, for $\varepsilon < \varepsilon_o$ and for $\varepsilon > \varepsilon_o$.

Depicted s.E.d. undoubtedly demonstrate the advantage of two-head detector utilization on two-track interfering channel. Over the range of ITI values,

$$0 < \varepsilon < \varepsilon_o = 0.293, \quad (14)$$

TTTH s.E.d. gradually increases. In this range the growth of 7.18% can be noticed, which additionally helps two-head detector to successfully combat channel ITI distortions.

**Case c** Previous analysis has shown that pairs $(e_{1}, 0)$ and $(e_{1}, -e_{1})$ have large influence on TTTH s.E.d. and potentially they can be the pair that, for some particular in-track input error $e_{1}$, defines the TTTH m.s.d. But, there is a question, is there any other two-track input error sequence pair $(e_{1}, e_{2})$ that produces lower s.E.d. than the one determined in the Case a) and Case b). In the following analysis this problem answer is presented.

Define the following two functions

$$R = d_{2D}^2(e_{1}, e_{2}) - d_{2D}^2 a, \quad Q = d_{2D}^2(e_{1}, e_{2}) - d_{2D}^2 b, \quad (15)$$

and search for the two-track input error sequence pairs $(e_{1}, e_{2})$, for which the $R < 0$ and $Q < 0$, corresponding to the conditions

$$d_{2D}^2(e_{1}, e_{2}) < d_{2D}^2 a, \quad (16)$$

These two functions depend on ITI level, so the function $R$ has nulls for ITI levels

$$e_{1,2} = \frac{2 S}{d_2} \pm \sqrt{\left(\frac{2 S}{d_2}\right)^2 - 1}, \quad (17)$$

where $d_2 = d_2^2(e_{1})$ and $S$ is

$$S = \sum_{i=-\infty}^{\infty} e_{1,2}(D)D^i. \quad (18)$$

One of the critical cases for TTTH s.E.d., depicted in Fig. 3, appears when the function $R$ lower null satisfy condition

$$0 < \frac{2 S}{d_2} - \sqrt{\left(\frac{2 S}{d_2}\right)^2 - 1} < \varepsilon_o, \quad (19)$$
which correspond to the
\[
\frac{2S}{d_2^2} > \frac{\epsilon_0^2 + 1}{2\epsilon_0} = 2.
\] (20)

Of course function \( R \) lower null exist when the condition
\[
\frac{2S}{d_2^2} > 1,
\] (21)
is fulfilled, which also correspond to the request \( R < 0 \).

Function \( Q \) possess nulls for ITI level
\[
\epsilon_{d12} = \frac{2S - 2d_1^2}{d_1^2 - d_2^2} ± \sqrt{\frac{(2S - 2d_1^2)^2}{d_1^2 - d_2^2} - 1},
\] (22)
where \( d_1^2 = d_1^2(\epsilon_0) \). The existence of the function \( Q \) nulls drives to the two possibilities, the first
\[
d_1^2 > d_2^2 : \frac{2S}{d_1^2} < 1 + \frac{d_2^2}{d_1^2} \wedge \frac{2S}{d_1^2} > 3 - \frac{d_2^2}{d_1^2},
\] (23)
and the second
\[
d_1^2 < d_2^2 : \frac{3 - \frac{d_2^2}{d_1^2} < \frac{2S}{d_1^2} < 1 + \frac{d_2^2}{d_1^2}.
\] (24)

The paper analyse TTTH ITI-based channel model with ITI level \( \epsilon > 0 \), so the bottom of the function \( Q \) has to be on the right side of \( y \)-axis. This means that condition
\[
-\frac{2S - 2d_1^2}{d_2^2 - d_0^2} > 0.
\] (25)
have to be satisfied. Condition (25) leads to the two possibilities, the first
\[
d_1^2 > d_2^2 : \frac{2S}{d_1^2} < 2,
\] (26)
and the second
\[
d_1^2 < d_2^2 : \frac{2S}{d_1^2} > 2.
\] (27)

Combination of (23), (24), (26) and (27) produces following conditions for function \( Q \) nulls existence and function \( Q \) bottom position
\[
d_1^2 > d_2^2 : \frac{2S}{d_1^2} < 1 + \frac{d_2^2}{d_1^2} \wedge \frac{2S}{d_1^2} > 3 - \frac{d_2^2}{d_1^2},
\] (28)

Another one of the critical cases for TTTH s.E.d. appears when the function \( Q \) lower null satisfy condition
\[
-\frac{2S - 2d_1^2}{d_1^2 - d_2^2} ± \sqrt{\frac{(2S - 2d_1^2)^2}{d_1^2 - d_2^2} - 1} > \epsilon_g,
\] (29)
which leads to the two possibilities, first
\[
d_1^2 > d_2^2 : \frac{2S}{d_1^2} > 0.146 + 1.854 \frac{d_2^2}{d_1^2},
\] (30)
and the second
\[
d_1^2 < d_2^2 : \frac{2S}{d_1^2} < 0.146 + 1.854 \frac{d_2^2}{d_1^2}.
\] (31)

Combination of the condition (21), (28), (30) and (31) leads to the search criterion
\[
d_1^2 > d_2^2 \wedge 0.146 \cdot d_1^2 + 1.854 \cdot d_2^2 < 2S < d_1^2 + d_2^2,
\] (32)
which enables to find two-track input error sequence pairs \( (\epsilon_1, \epsilon_2) \) that produce lower s.E.d. than the one determined in the Case a) and Case b), shown in Fig. 4.

Existence of two-track input error sequence pairs \( (\epsilon_1, \epsilon_2) \) which produce Case c) TTTH s.E.d. negatively reflects to the two-head channel detector, decreasing its robustness for ITI distortions.

IV. Case c) E"PR4 TTTH s.E.d. SEARCH RESULTS

Computer search for Case c) critical two-track input error sequence pairs \( (\epsilon_1, \epsilon_2) \), is based on search criterion (32). Table I list a sample of two-track input error sequence pairs for E"PR4 TTTH channel.

<table>
<thead>
<tr>
<th>( (\epsilon_1, \epsilon_2) )</th>
<th>( (d_1^2, d_2^2) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( [1 1 1 -1 1 0 0] )</td>
<td>d_1^2 = 72</td>
</tr>
<tr>
<td>( [1 1 1 1 1 0 0] )</td>
<td>d_1^2 = 24</td>
</tr>
<tr>
<td>( [1 1 1 -1 1 0 0] )</td>
<td>d_1^2 = 92</td>
</tr>
<tr>
<td>( [1 1 1 1 1 0 0] )</td>
<td>d_1^2 = 24</td>
</tr>
<tr>
<td>( [1 1 1 1 1 0 0] )</td>
<td>d_1^2 = 92</td>
</tr>
<tr>
<td>( [1 1 1 1 1 0 0] )</td>
<td>d_1^2 = 40</td>
</tr>
<tr>
<td>( [1 1 1 1 1 1 0] )</td>
<td>d_1^2 = 92</td>
</tr>
<tr>
<td>( [1 1 1 1 1 1 0] )</td>
<td>d_1^2 = 40</td>
</tr>
<tr>
<td>( [1 1 1 1 1 1 0] )</td>
<td>d_1^2 = 112</td>
</tr>
<tr>
<td>( [1 1 1 1 1 1 0] )</td>
<td>d_1^2 = 44</td>
</tr>
<tr>
<td>( [1 1 1 1 1 1 1] )</td>
<td>d_1^2 = 112</td>
</tr>
<tr>
<td>( [1 1 1 1 1 1 1] )</td>
<td>d_1^2 = 44</td>
</tr>
<tr>
<td>( [1 1 1 1 1 1 1] )</td>
<td>d_1^2 = 104</td>
</tr>
<tr>
<td>( [1 1 1 1 1 1 1] )</td>
<td>d_1^2 = 44</td>
</tr>
<tr>
<td>( [1 1 1 1 1 1 1] )</td>
<td>d_1^2 = 104</td>
</tr>
<tr>
<td>( [1 1 1 1 1 1 1] )</td>
<td>d_1^2 = 28</td>
</tr>
<tr>
<td>( [1 1 1 1 1 1 1] )</td>
<td>d_1^2 = 108</td>
</tr>
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<td>d_1^2 = 28</td>
</tr>
<tr>
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<td>d_1^2 = 108</td>
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</tr>
</tbody>
</table>
V. E\textsuperscript{2}PR4 TTTH channel m.s.d.

Analysed s.E.d. (10), (12) and (14) depends on particular error sequence \( e_{c1} \). Therefore, the classification of input error sequences for the E\textsuperscript{2}PR4 in-track channel, according to their length and distance is essential for E\textsuperscript{2}PR4 TTTH m.s.d. determination. Table II list a sample of input error sequences for E\textsuperscript{2}PR4 channel.

<table>
<thead>
<tr>
<th>( e_{c1} )</th>
<th>( d_{21} (e_{c1}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10</td>
</tr>
<tr>
<td>1-1+1</td>
<td>6</td>
</tr>
<tr>
<td>1-1+1-1</td>
<td>8</td>
</tr>
<tr>
<td>1-1+1-1(1-1)</td>
<td>8</td>
</tr>
</tbody>
</table>

It can be seen that error sequence \( \pm [1 -1 1] \) produces in-track m.s.d.

\[
d_{\text{min}}^2 = d^2(\pm [1 -1 1]) = 6,
\]

which directly reflects to the Case a) and Case b) E\textsuperscript{2}PR4 TTTH m.s.d. versus ITI level

\[
d_{2D\text{min}}^2 = (1 + \varepsilon^2) \cdot d_{\text{min}}^2 \quad 0 < \varepsilon < \varepsilon_{c1},
\]

\[
d_{2D\text{min}}^2 = 2 \cdot (1 - \varepsilon^2) \cdot d_{\text{min}}^2 \quad \varepsilon_{c1} < \varepsilon < 2 + \sqrt{3}.
\]

From the first condition of Case c) search criteria (32), it can be seen that neither one of in-track input error sequence, in the second track, does not posses s.E.d. lower then in-track m.s.d. (33). Therefore E\textsuperscript{2}PR4 TTTH channel m.s.d. is determined with \( \pm [1 -1 1] \) error pattern, and is depicted in Fig. 6.

For E\textsuperscript{2}PR4 in-track equalization of TTTH channel there is no effect of lowering TTTH m.s.d. around ITI level \( \varepsilon_{c1} \), as shown in Fig. 5. Case c) search criteria (32), does not result in any two-track input error sequence of the form \( \pm [1 -1 1] \), \( e_{c2} \) which can decrease E\textsuperscript{2}PR4 TTTH m.s.d., as shown in Fig. 6. But, when distance enhancing coding techniques, such [13], [14], were used for E\textsuperscript{2}PR4 TTTH channel, Case c) search criteria will be valuable for distance analysis and detector performance improvement.

VI. CONCLUSION

Square Euclidean distance analysis is one of the most valuable techniques for channel feature investigation. Based on its results many coding techniques are and can be developed for channel detector performance improvement. In this paper distance properties of two-track two-head channel were analysed, focused on E\textsuperscript{2}PR4 in-track equalization. Valuable search criteria for finding two-track critical input error sequences, which possible negatively effect to the TTTH s.E.d. were presented. This search criterion will be helpful in distance analysis utilisation of distance enhancing coding technique over TTTH channel.

**REFERENCE**


