
PRECISE MEASUREMENTS OF LASER BEAM RADIUS

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I. INTRODUCTION

Measurement of laser beam radius using a small aperture photodiode hides several sources of error that are considered in this paper. Standard small area photodiode is housed in metal box with glass window. Removing glass window eliminates error caused by reflection and stray radiation due to dispersion, but still remains problem of aperture size. In this paper we investigate electromagnetic field propagated through an arbitrary small hole in front of the photodiode. As a next step we calculate the error due to finite ratio of measuring aperture and the size of the beam at the measuring plane.

II. TRANSMISSION OF ELECTROMAGNETIC FIELD THROUGH A PINHOLE

We consider a circular photodiode of radius \(a_d\) and a pinhole of radius \(a\), as shown in Fig.1. The pinhole is moved across the plane at \(z=z_1\) so that the beam axis and the photodiode system axis remain parallel.

![Fig.1. Geometry of laser – photodiode arrangements](image)

The electromagnetic field radiated through the pinhole can be calculated starting from the field in the aperture plane in polar coordinates \([1]\)

\[
E_{ap}(r, \phi, z_1) = \frac{W_0E_0}{W_{z_1}} e^{-b r^2} e^{-j(kz_1 - \Psi(z_1))} \tag{1}
\]

where

\[
b = \frac{1}{W_{z_1}^2} - j\frac{\pi}{R_{z_1} \lambda}
\]

\[
w_{z_1} = w_0 \sqrt{1 + (\lambda z_1 / \pi w_0^2)^2}
\]

\[
R_{z_1} = z_1 \left[1 + (\pi w_0^2 / \lambda z_1)\right]
\]

Calculation of the field across the photodiode area is needed to investigate whether all the power incident on the aperture reaches photodiode area. To do this we have to find field behind the pinhole at the distance equal to separation between the pinhole and the photodiode area. As the distance between the hole and photodiode sensitive area is small we have to apply spectrum of plane wave technique. In this case we find first the spectrum of plane waves for the known aperture distribution and then calculate the field from the integral \([1]\)

\[
E(r, \phi, z) = \frac{w_0E_0}{W_{z_1}} \int_0^{\infty} \left[ e^{-k^2 r^2} J_0(k r) e^{j k r} \right] d k
\]

\[
\times \int_0^{\infty} J_0(k r) e^{-\frac{k^2}{2}} k d r
\]

For example, the Poynting vector of the field behind a pinhole of radius \(a\) situated at \(z_1 = 800\) mm for \(w_0 = 0.5\) mm, wavelength \(\lambda = 0.0006328\) mm; \(z_n = 5\) mm looks like shown in Fig.2.

![Fig.2 Field behind a pinhole of radius 0.25 mm](image)

The incident power density, integrated over the area of photodiode of radius \(a_d\) centered at the point \((0, y_c, z_n)\), is the received power that could be calculated from the integral of the field intensity over the photodiode area. However, as this
is a complex problem we can indirectly treat this by considering the extent of the field as calculated from eq.(2). From the figure we see that the square of the field is practically zero for \( a > 0.35 \) mm, and therefore we can consider that all power falls on the photodiode.

III. ERROR IN MEASUREMENTS DUE TO FINITE PIN HOLE SIZE

When the measurements of \( w_z \) are made with a pinhole of finite size in front of the photo detector, a systematic error is introduced because detector subtends part of the beam rather than a point field. However, this can be corrected by calculation of power received by the detector as compared with the same power that would be received by an infinitely small size detector.

![Fig.3 Coordinate system used in beam measurement](image)

We assume that the Gaussian beam across the measuring plane at \( z = z_1 \) is expressed by the equation (see Fig.3)

\[
|E(x, y)| = E_z e^{-\left(\frac{x^2 + y^2}{w_z^2}\right)}
\]  

(3)

and we want to measure \( w_z \). Let the pinhole have circular shape of radius \( a \). The power received by this detector at the point \((x_0, y_0)\) can be calculated from the integral

\[
P_d(x_0, y_0) = \int_{-a-x_0}^{a-x_0} \int_{-y_0}^{y_0} \left|E_z(x, y)\right|^2 e^{-\frac{x^2+y^2}{w_z^2}} \, dx \, dy
\]

(4)

At the point \((0,0)\) the above integral can be solved in a closed form. In this case we use the polar coordinates since the integration with respect to \( r \) and \( \phi \) are independent:

\[
P_d(0,0) = \frac{|E_z(0,0)|^2}{2Z_0} \int_0^{2\pi} d\phi \int_0^a e^{-\frac{r^2}{w_z^2}} r \, dr
\]

\[
= \frac{|E_z(0,0)|^2}{2Z_0} \frac{2\pi w_z^2}{4} \left(1 - e^{-\frac{a^2}{w_z^2}}\right)
\]

(5)

The ratio of the two calculated received power

\[
R_d(x_0, y_0) = \frac{P_d(x_0, y_0)}{P_d(0,0)}
\]

(6)

is dependent on the ratio \( w_z / a \), and in the limiting case when \( a \to 0 \) it will be given by

\[
R_{d(a\to0)}(x_0, y_0) = e^{-\frac{x_0^2+y_0^2}{w_z^2}}
\]

(7)

Since \( w_z \) is a constant, then from (7) we can write

\[
x_0^2 + y_0^2 = \frac{w_z^2}{2} \ln \left[R_{d(a\to0)}(x_0, y_0)\right]
\]

(8)

If the shape of \( R_d(x_0, y_0) \) is not much changed from the Gaussian shape, we can assume that a similar expression to (8) can be written from eq. (7) with the new value of \( w_{\text{zm}} \), i.e.

\[
x_0^2 + y_0^2 = \frac{w_{\text{zm}}^2}{2} \ln \left[R_d(x_0, y_0)\right]
\]

(9)

By dividing (8) and (9), after simple calculations we obtain

\[
\frac{w_z^2}{w_{\text{zm}}^2} = \sqrt{\frac{\ln \left[R_{d(a\to0)}(x_0, y_0)\right]}{\ln \left[R_d(x_0, y_0)\right]}}
\]

(10)

Calculation of integral (4) has been done with a MATCAD program and typical results are shown in Fig.4.
For measured \( w_{zm}/a \), using the graph in Fig.4 we find the correction factor with which we have to multiply the measured value to obtain the corrected \( w_z \).

Fig.5. Percent systematic error in measurements of \( w_{zm} \)  
(Crosses are calculated points)

Fig.5. Shows systematic error in measurements of \( w_{zm} \). For \( w_z/a \geq 7 \), the error will be less than 1%. The error increases rapidly for \( w_z/a \leq 4 \). Measured values are always greater than the true values.

Fig.6 is the plot of \( \ln \left[ R_y(x_0, y_0) \right] \) vs. \( x_0 \) \((y_0 = 0)\) for three different ratios \( w_z/a \). The \( w_{zm} \) are determined at the ordinate level equal to \(-2\). The ratio \( w_z/w_{zm} \) is close to unity for \( w_z/a = 10 \), and the measured \( w_{zm} \) is only 0.5% higher than the true \( w_z \). It is interesting that the shapes of the curves differ very little from the straight line, even when \( w_z/a = 1.5 \). In the latter case the error is nearly 23%. However, for \( w_z/a \), distortion is noticeable as can be seen from Fig.5 and the error rises to about 50%.

Fig.7. Logarithmic vs. linear \( x_0^2 \) scale for different \( w_z/a \):  
full line \( w_z/a = 10 \); short dashed \( w_z/a = 4 \); long dashed \( w_z/a = 1.5 \)
IV. CONCLUSIONS

Measurements of laser beam characteristics, at relatively small distances can be improved by using a pinhole in front of a photodiode of large size that, if used directly as the detector, would produce considerable error in determination of the beam radius $w_z$. In this paper correction curves are obtained which can give correct beam radius if the size of pinhole is known.

REFERENCES


Abstract

Precise measurement of laser beam radius requires very small area photodiode or feeding of the photodiode through a small pinhole. In this paper we have calculated electromagnetic field behind a pinhole of arbitrary size and the remaining error introduced by finite pinhole size. For known pinhole size, true beam radius is obtained from measured beam radius using calculated correction term.

Presizno merenje radijusa laserskog snopa

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