CALCULATION OF GAUSSIAN PULSE PASSING THROUGH
FIBERS WITH POSITIVE AND NEGATIVE DISPERSION

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I INTRODUCTION
Transmission of Gaussian pulse in a coherent optical system through a fiber with uniform dispersion is well known and can be found, for example, in [1]. In this paper we derive expressions for the transmission of Gaussian pulse through fiber of length $L_1$ with positive dispersion, and of length $L_2$ with negative dispersion. The lengths of the two fibers are selected in such a way that the total dispersion is zero at the operating wavelength.

II THEORY
Analytical signal of Gaussian pulse modulating coherent carrier is given by

$$e_{in}(t) = E_0 e^{-rac{t^2}{2\sigma_0^2}} e^{j\Omega_0 t},$$

where $\sigma_0$ is proportional to pulse width and $\Omega_0$ is the carrier frequency. The Fourier transform of (1) is the spectrum for positive frequencies

$$E_{in}(\Omega) = \sqrt{2\pi}\sigma_0 E_0 e^{-(\Omega - \Omega_0)\sigma_0^2/2}$$

Assuming linear optical fiber, we can write transfer function in the form

$$H_F(\Omega, L) = e^{-(\alpha + j\beta)L}$$

where $\alpha$ and $\beta$ are attenuation and phase coefficients, respectively. As the fiber attenuation is practically constant in the bandwidth required by the modulated carrier, we will not take it into account as it does not affect the shape of the output signal. Since we consider two fibers of various phase coefficient characteristics, we will write our combined transfer function in the form

$$H_F(\Omega, L_1 + L_2) = e^{-j(\beta_1 L_1 + \beta_2 L_2)}$$

where we assume expansions of the phase coefficients for positive frequencies around carrier frequency $\Omega_0$

$$\beta_i(\Omega) = \beta_{0i} + \beta_{1i}^0(\Omega - \Omega_0) + \beta_{2i}^0(\Omega - \Omega_0)^2 / 2 + \beta_{3i}^0(\Omega - \Omega_0)^3 / 6, \quad i = 1, 2.$$  

By taking the inverse Fourier transform of the functions given by (2) and (4) we arrive at the integral for the output signal [1]:

$$e_{out}(t) = \frac{E_0\sigma_0 e^{\frac{\sigma_0^2}{2}}}{\sqrt{2\pi}} \cdot \int_{-\infty}^{\infty} \exp\left[-\frac{z^2}{2} - \frac{1}{2} \left(\beta_{01} L_1 + \beta_{02} L_2\right) z + \left(\beta_{01}^0 L_1 + \beta_{02}^0 L_2\right) z^2/2 + \left(\beta_{01}^0 L_1 + \beta_{02}^0 L_2\right) z^3/6\right] + jzt \, dz$$

where $z = \Omega - \Omega_0$. If we neglect the cubic term, integral (6) can be solved. After simplification, (6) can be reduced to form

$$e_{out}(t) = C \int_{-\infty}^{\infty} \exp\left[-\frac{\sigma_0^2}{2} + \frac{1}{2} \left(\beta_{01}^0 L_1 + \beta_{02}^0 L_2\right) z^2\right] + j\left[\left(\beta_{01}^0 L_1 + \beta_{02}^0 L_2\right) z\right] \, dz$$

where

$$C = \frac{E_0\sigma_0 e^{\frac{\sigma_0^2}{2}}}{\sqrt{2\pi}}.$$  

Exponential term in (7) can be rewritten as

$$-\left[Dz + \frac{B}{2D}\right]^2 - \frac{B^2}{4D^2}$$
By introducing a new variable \( D = Dz + j \frac{B}{2D} \), integral (7) can be solved and the final result is given by Eq. (9) (see above), where

\[
\sigma_{L_1+L_2} = \sigma_0 \left[ 1 + \left( \frac{\beta_{02}^2 L_1 + \beta_{02}^2 L_2}{\sigma_0^2} \right)^2 \right]^{1/2},
\]

and

\[
\Psi = \frac{1}{2} \tan^{-1} \left( \frac{\beta_{02}^2 L_1 + \beta_{02}^2 L_2}{\sigma_0^2} \right).
\]

### III CALCULATION OF PHASE COEFFICIENTS

Phase coefficient for step-index fiber is obtained from

\[
\beta(\Omega) = \sqrt{k_0^2 n_{core}^2 - U^2 / a^2}.
\]

Here, \( k_0 = \frac{2\pi}{\lambda} \), \( n_{core} \) is the refractive index of the core, \( a \) is the core radius and \( U \) is the solution of the characteristic equation

\[
\frac{UJ_1(U)}{J_0(U)} = \frac{WK_1(W)}{K_0(W)}.
\]

where \( U \) and \( W \) are related through normalized frequency, \( \nu^2 = U^2 + W^2 = 2k_0^2 a^2 \Delta \), which can also be expressed as \( V = \frac{a\sqrt{2\Delta}}{c} \Omega \), where \( c \) is the speed of light in vacuum and \( \Omega \) is optical frequency.

The core radius is obtained from selected cutoff wavelength and relative difference of the core and cladding refractive indices

\[
a = \frac{2.405\lambda_c}{2\pi \sqrt{2\Delta}}.
\]

To find series expansion as expressed in (5), we calculate a series of \( \beta(\Omega) \) from Eq. (10) in the range \( \lambda_{\min} < \lambda < \lambda_{\max} \), corresponding to \( \Omega_{\max} > \Omega > \Omega_{\min} \) and look for the coefficients of the best fitting polynomial

\[
\beta_i(\Omega) = a_i + b_i \Omega + c_i \Omega^2 + d_i \Omega^3, \quad i = 1, 2 \quad . (13)
\]

Now we have to find coefficients as in Eq.(5). This can be obtained from the matrix equation [2]

\[
\begin{bmatrix}
\beta_1 & 1 & \Omega_0 & \Omega_0^2 & \Omega_0^3 \\
\beta_1^{(1)} & 0 & 1 & 2\Omega_0 & 3\Omega_0^2 \\
\beta_1^{(2)} / 2 & 0 & 0 & 1 & 3\Omega_0 \\
\beta_1^{(3)} / 6 & 0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
a \\
b \\
c \\
d
\end{bmatrix}
= \Omega_0. \quad (14)
\]

### IV NUMERICAL EXAMPLE

As an illustration of numerical results of the above developed theory we selected fiber No.1 with positive dispersion which is close to standard monomode fiber with \( \Delta = 0.45\% \) and its parameters are calculated based on data in [3]. Zero dispersion for this fiber is 1.35 \( \mu \)m with cutoff wavelength 1.28 \( \mu \)m, resulting in the core radius \( a = 3.64 \mu \)m. At the selected operating frequency 1.55 \( \mu \)m, total dispersion is 12.71 ps/(nm km). The fiber No.2, with negative dispersion is again selected from data given in [3] and has \( \Delta = 1.48\% \), cutoff wavelength 0.8 \( \mu \)m, with calculated \( a = 1.23 \mu \)m and dispersion at the operating frequency \(-57.22 \text{ ps/(nm km)}\). The calculated parameters of the fibers are given in Tab. 1.
**Tab. 1.** Fiber parameters.

<table>
<thead>
<tr>
<th>Fiber No.</th>
<th>$\beta_1$</th>
<th>$\beta_1^{(1)}$</th>
<th>$\beta_1^{(2)}$</th>
<th>$\beta_1^{(3)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>No.1</td>
<td>5.864132 $\times 10^6$</td>
<td>4.89684 $\times 10^5$</td>
<td>-80.99969</td>
<td>14.36912</td>
</tr>
<tr>
<td>No.2</td>
<td>5.866491 $\times 10^6$</td>
<td>4.90996 $\times 10^5$</td>
<td>364.6756</td>
<td>-6.093772</td>
</tr>
</tbody>
</table>

The fiber lengths are calculated from the condition of zero total dispersion at the operating frequency expressed through the equation

$$\beta_1^{(2)} L_1 + \beta_2^{(2)} L_2 = 0 \quad (15)$$

For selected total length of the two fibers of $L_1 + L_2 = 70$ km, the length of the first mentioned fiber is $L_1 = 57.278$ km and that of the second $L_2 = 12.722$ km. Table 1 gives calculated coefficients for the two considered fibers. The dispersion curves in ps/(nm km) are shown in Fig.1, as the functions of wavelength.

**Fig. 1.** Dispersion curves for fiber No.1: $d(1550$ nm) = 12.71 ps/(nm km) and for fiber No.2.: $d(1550$ nm) = -57.22 ps/(nm km).

If the lengths are not as calculated, then the system acts as having dispersion which can be found from the relation

$$d_{eq} = \frac{d_0 L_1 + d_0 L_2}{L_1 + L_2}. \quad (16)$$

In Tab. 2 several values of $d_{eq}$ have been given as an illustration.

The result of numerical integration of integral (7) is shown in Fig.2, for several pulse widths at very low dispersion. For pulse of width 1 ps,

**Fig. 2.** Calculated output signals for very low dispersion – in that case, the integral must include the cubic term. Pulse width is 1 ps.

In order to see more generaly the behaviour of the system for several dispersion produced by different combination of fiber lengths, on Figs. 4 and 5 the curves $\sigma_i(\sigma_0)$ and $f_{0e}(\sigma_0)$ are plotted.

**Fig. 3.** Input pulse width is 3 ps – cubic term has small effect. Attainable bit rate is 100 Gb/s.

In Tab. 2. Fiber length mismatch effects.

<table>
<thead>
<tr>
<th>$L_1$ (km)</th>
<th>$L_2$ (km)</th>
<th>$d_{eq}$ (ps/nm km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>56</td>
<td>14</td>
<td>-1.277</td>
</tr>
<tr>
<td>57</td>
<td>13</td>
<td>-0.278</td>
</tr>
<tr>
<td>57.3</td>
<td>12.7</td>
<td>0.022</td>
</tr>
<tr>
<td>58</td>
<td>12</td>
<td>0.721</td>
</tr>
<tr>
<td>59</td>
<td>11</td>
<td>1.72</td>
</tr>
</tbody>
</table>

The result of numerical integration of integral (7) is shown in Fig.2. for several pulse widths at very low dispersion. For pulse of width 1 ps,
Fig. 4. Output pulse width vs. input pulse width for several combinations of fiber lengths.

Fig. 5. Bit rate vs. input pulse width: maximum bit rate is achieved for $\sigma_i = \sqrt{2}\sigma_0$.

V CONCLUSION
Combination of fibers with positive and negative dispersions has been used to reduce the effect of excessive total dispersion at the operating frequency far from the zero dispersion frequency. We have shown that certain departure in fiber lengths from that when the overal dispersion is zero can be tolerated although the reduction of maximum fiber bit rate is very pronounced. The developed theory enables analysis of numerous combinations and fast numerical calculations.

REFERENCES

PROSTIRANJE GAUSSOVOG IMPULSA KROZ VLAKNA SA POZITIVNOM I NEZATIVNOM DISPERZIJOM
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