AN APPROXIMATIVE MODEL FOR ANALYZING VERTICAL ANTENNAS ABOVE A GROUND PLANE

Rančić, M., Rančić, P.
Faculty of Electrical Engineering in Niš

I INTRODUCTION

One of the research results considering the influence of the ground conductivity on the characteristics of wire structures placed above ground, which is expressed by the Sommerfeld's Integral Kernel – SIK (1909., [1]), is presented in this paper. A new simple model for the SIK is proposed. Besides simplicity, the model should be general and should give output numerical results (for the unknown current distribution - UCD, for the input impedance/admittance, near and far EM field) of satisfying accuracy.

The results and previous experiences obtained while solving the problem of the vertical (VDA) and horizontal (HDA) dipole antennas in the presence of a lossy half-space ([2] - [5] and [6]), were used in this paper. Besides simplicity and satisfying accuracy of the output numerical results, the models for the SIK proposed in this paper and the one proposed in [6], also have the general character in contrast to those in [2]-[5]. This is because no limitations for the values of the electrical parameters of the ground, i.e. the refraction index earth/air, were introduced.

These characteristics of the model (simplicity, generality and accuracy) are illustrated in this paper by numerous numerical experiments, and the results for the input impedance/admittance are compared with the previously published ones ([2], [7] - [9]).

II THEORETICAL OUTLINE OF THE MODEL

II.1 DESCRIPTION OF THE MODEL GEOMETRY

The vertical asymmetrical dipole antenna (VDA) in the presence of the imperfect ground plane is considered. The antenna conductors are linear of length l_1 (upper) and l_2 (lower), and of equivalent circular cross-section with the radius $a_k \ll l_k$ and $a_k \ll \lambda_0$, $k = 1,2$ (λ_0 - wave-length in the air). The antenna is placed above the ground that can be treated as a homogenous and isotropic medium. Electric parameters of the air $\sigma_0 = 0$, ε_0, μ_0, and of the soil σ_1, ε_1, μ_1 (σ_i - conductivity; $\varepsilon_i = \varepsilon_{ri}\varepsilon_0$ - permittivity; $\mu_i = \mu_0$ - permeability, and $\underline{\sigma}_i = \sigma_i + j\omega\varepsilon_i$ - complex conductivity;

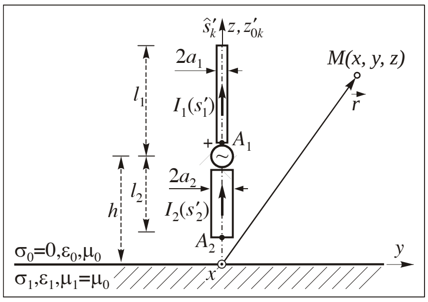

Figure 1: Schematic illustration of the vertical asymmetrical dipole antenna above a homogenous and isotropic medium.

$\underline{\gamma}_i = \alpha_i + j\beta_i = (j\omega\mu_i\underline{\sigma}_i)^{1/2}$, $i = 0,1$ - complex propagation constant; $\omega = 2\pi f$ - angular frequency; and $\underline{n}_{10} = \underline{\gamma}_1/\underline{\gamma}_0 = \underline{\varepsilon}_{r1}^{1/2} = (\varepsilon_{r1} - j60\sigma_1\lambda_0)^{1/2}$ - the refraction index) are known.

The antenna feeding is approximated by an ideal voltage generator of voltage U and of frequency *f*. The antenna feeding points are at height h, $h \ge l_2$. The UCD is denoted by $I_k(s'_k)$, $0 \le s'_k \le l_k$, $k = 1,2$. Beginnings of s'_k - axes are at the points $z_{A1} = h$, and $z_{A2} = h - l_2$ (i.e. $z'_{0k} = z_{Ak} + s'_k$, $k = 1,2$). The illustration of the system is given in **Figure 1.**

II.2 THE HERTZ'S VECTOR POTENTIAL

The Hertz's vector potential, $\vec{\Pi}_0 = \Pi_{s_n 0}\,\hat{z}$, which originates from the UCD localized along the VDA conductors, is given in the following form:

$$\Pi_{s_n 0} = \frac{1}{4\pi\underline{\sigma}_0}\sum_{k=1}^{2}\int_{s'_k=0}^{l_k} I_k(s'_k)\left[K_0(r_{1k}) + S_v(r_{2k})\right]\mathrm{d}s'_k, \quad (1)$$

where: $K_0(r_{ik}) = \exp(-\underline{\gamma}_0 r_{ik})/r_{ik}$, $i,k = 1,2$ is the potential kernel of the source and its image, r_{ik} - distance from the source, i=1, and its image, i=2, to the field point in air, i.e. $r_{1k} = [\rho'^2_{0k} + (z - z'_{0k})^2]^{1/2}$, $r_{2k} = [\rho'^2_{0k} + (z + z'_{0k})^2]^{1/2}$, $\rho'_{0k} = [(x - x'_{0k}) + (y - y'_{0k})^2]^{1/2}$; $S_v(r_{2k})$ is the SIK of the form

$$S_v(r_{2k}) = \int_{\alpha=0}^{\infty} \tilde{R}_{z10}\,\tilde{K}_0(r_{2k})\,\mathrm{d}\alpha =$$

$$= \int_{\alpha=0}^{\infty}\left(\frac{\underline{n}_{10}^2 u_0 - u_1}{\underline{n}_{10}^2 u_0 + u_1}\right)\left(\frac{e^{-u_0(z+z'_{0k})}}{u_0}\alpha\,J_0(\alpha\rho'_{0k})\right)\mathrm{d}\alpha, \quad (2)$$

where $\tilde{R}_{z10}$ and $\tilde{K}_0(r_{2k})$ are the reflection coefficient (TRC) and the standard potential kernel in the transformed domain of the variable α, respectively, i.e.:

$$\tilde{R}_{z10} = (\underline{n}_{10}^2 u_0 - u_1)/(\underline{n}_{10}^2 u_0 + u_1), \quad (3)$$

$$K_0(r_{2k}) = \int_{\alpha=0}^{\infty}\tilde{K}_0(r_{2k})\,\mathrm{d}\alpha, \quad (4)$$

$u_i = (\alpha^2 + \underline{\gamma}_i^2)^{1/2}$, $i = 0,1$ and $J_0(\alpha\rho'_{0k})$ is zero order of the first kind Bessel's function.

II.3 THE APPROXIMATE SOLUTION FOR THE SIK

In the paper [2], a simplified expression for the SIK, obtained under an assumption $n_{10} \gg 1$, was used for solving the problem of the VDA. Namely, if $n_{10} \gg 1$ and $u_1 \approx \underline{\gamma}_1$, the TRC, given by (3), is developed into the geometry progression series, and only the two first terms are adopted, i.e.

$$\tilde{R}_{z10}(u_0) = 1 - \tilde{T}_{z01} = 1 - \frac{2u_1}{\underline{n}_{10}^2 u_0 + u_1} \cong 1 - \frac{2}{\underline{n}_{10}} \frac{\underline{\gamma}_0}{u_0} . \quad (5)$$

Substituting (5) into (2), the SIK becomes

$$S_v(r_{2k}) \cong K_0(r_{2k}) - \frac{2\underline{\gamma}_0}{\underline{n}_{10}} \int_{v=s'_k}^{\infty} K_0(r_{2k}) \mathrm{d}v , \quad (6)$$

where $r_{2k} = [\rho'^2_{0k} + (z + z'_{Ak} + v)^2]^{1/2}$.

The numerical experiments have shown that the output numerical results of satisfying accuracy are obtained if $n_{10} > 3$ ([2], [3], [4]).

For the purpose of solving the problem of the HDA, developments of the TRCs, $\tilde{R}_{z10}(u_0)$ and $\tilde{R}_{\eta 10}(u_0)$, into the Taylor's series around $u_0 = \underline{\gamma}_0$ are proposed in [5]. It is enough to adopt only the first three terms of the obtained series in order to simply and efficiently solve the SIK. By applying these models, the limitations for the n_{10} are lightened.

Using the good sides of both approximations (simplicity, efficiency and accuracy), a new simple approximation for the $\tilde{R}_{z10}(u_0)$ is proposed in this paper and also in [6]:

$$\tilde{R}_{z10}(u_0) \cong B + A(\underline{\gamma}_0 / u_0) , \quad (7)$$

where A and B are the unknown constants that are obtained for the adoption conditions $u_0 = \underline{\gamma}_0 / (\underline{n}_{10}^2 + 1)^{1/2}$ and $u_0 \to \infty$.

The constants A and B are of the following values:

$$B = (\underline{n}_{10}^2 - 1)/(\underline{n}_{10}^2 + 1) , \quad A = -B/(\underline{n}_{10}^2 + 1)^{1/2} . \quad (8)$$

Substituting (7) into (2), the SIK becomes

$$S_v(r_{2k}) \cong B[K_0(r_{2k}) - \frac{\underline{\gamma}_0}{(\underline{n}_{10}^2 + 1)^{1/2}} \int_{v=s'_k}^{\infty} K_0(r_{2k}) \mathrm{d}v] . \quad (9)$$

The constants A and B are derived in a different way in [6].

Analyzing (6) and (9), for the boundary values of the refraction coefficient $n_{10} = 1$ and $n_{10} \to \infty$, one can conclude:

- For $n_{10} = 1$ (the VDA in the free space), the SIK given by the approximation (9) gives an accurate solution, i.e. $S_v(r_{2k}) = 0$, while, with the approximation (6) an error is made since $S_v(r_{2k}) \neq 0$. This is because (6) is obtained under an assumption that $n_{10} \gg 1$;
- For $n_{10} \to \infty$ (the VDA in the presence of a perfect conducting plane), the SIK given by (9) primarily has the form (6) for $n_{10} \gg 1$, and when $n_{10} \to \infty$, the accurate solution for (9) is obtained, i.e.: $S_v(r_{2k}) = K_0(r_{2k})$;
- The integral with an infinite upper bound in (6) and (9) is numerically solved in the same way as in [2, Appendix A2].

From the previous analysis one can conclude that the approximation (9) is applicable for all combinations of $\varepsilon_{r1} \in [1, 81]$ and $\sigma_1 \in [0, \infty)$. This conclusion is confirmed by numerous numerical experiments given in this paper.

In order to obtain the output numerical results for the UCD and the input impedance/admittance, the system of integral equations of Hallen's type (SIE-H) was used. The point–matching method and the polynomial approximation for the UCD ([10], [11]) were used for numerical solving of the SIE-H.

III NUMERICAL RESULTS

Based on the proposed method and the adopted model for the SIK, necessary changes were made in the already developed program package ([2]) for numerical calculations on the PC.

The results for the input impedance of the VDA above a homogenous ground obtained by the proposed model and those from the other authors ([7], [8], [9]), are shown in **Tables 1** and **2**. Based on these results, it can be concluded that the proposed model gives the results of satisfying accuracy.

The results for the input resistance and reactance of the half-wave dipole placed above the homogenous ground ($\varepsilon_{r1} = 10$, $\sigma_1 = 3\,\mathrm{mS/m}$) as a function of the height h, are given in **Figure 2**. The solid line presents the results if the SIK is approximated by (9) and the dashed line, the case of the SIK model given in [6]. For the sake of comparing, the results obtained using the model (6) from [2] (squares) and those from [9] (circles), are shown in the same picture. Detailed comparing of the results for the model (6) with the results of numerous authors is given in [2].

The results for the input resistance and reactance of the half-wave dipole placed at height $h = l_2 = 0.25\lambda_0$, as a function of the equivalent conductance of the ground $\sigma_1\lambda_0$, and for different values of the relative permittivity ε_{r1} as a parameter (solid lines), are given in **Figure 3**. The results obtained by the model from [6] (dashed lines) are shown in the same picture.

The results for the input resistance and reactance of the half-wave dipole placed at height $h \geq l_2$, as a function of the equivalent conductance of the ground $\sigma_1\lambda_0$ and for different values of the height h as a parameter, are given in **Figure 4**. The relative permittivity is $\varepsilon_{r1} = 10$.

Based on the results given in **Tables 1** and **2** and **Figures 2** and **3,** it can be concluded that the proposed model, for the real parameters of the homogenous ground and $n_{10} > 3$, gives the results for the input resistance and reactance that slightly deviate from the ones obtained by the model in [2]. Besides this, the results obtained by the proposed new model have a general character, which can be concluded from the analysis given in section II.3 and presented numerical results for $1 \leq \varepsilon_{r1} < 10$ and $\sigma_1\lambda_0 < 0.01\,\mathrm{S}$.

Table 1: The input impedance, Z_a in $[\Omega]$, of the VDA above flat ground as a function of the height of its centre h: $l_1 = l_2 = 0.25\lambda_0$, $a_1 = a_2 = 5 \cdot 10^{-4}\lambda_0$, $f = 3\,\mathrm{MHz}$, $\varepsilon_{r1} = 10$, $\sigma_1 = 3\,\mathrm{mS/m}$, $n_{10} = 3.91 - j2.30$.

h/λ_0	Popović - Petrović [9]	SIE-TP [2] $M_1 = M_1 = 2$	This model $M_1 = M_1 = 2$
0.25	119.69+j 57.15	120.09+j 55.79	120.15+j 58.02
0.26	110.80+j 45.83	111.02+j 44.35	111.22+j 46.63
0.27	105.34+j 40.91	105.33+j 39.42	105.89+j 41.70
0.30	94.24+j 35.13	93.98+j 33.84	95.14+j 35.60
0.35	83.85+j 35.36	83.47+j 34.22	84.68+j 35.03

Table 2: The input admittance, Y_a in [mS], of the half-wave dipole as a function of the height h:

$l_1 = l_2 = 0.25\lambda_0$, $a_1 = a_2 = 0.007022\lambda_0$.

A: $f = 100\,\text{MHz}$, Sea water: $\varepsilon_{r1} = 80$, $\sigma_1 = 4\,\text{S/m}$, $n_{10} = 20.06 - j17.95$.

h/λ_0	Chang-Wait [7]	Chang-Wait [7] Mod. Col. Th.	Božilović [8]	SIE-TP [2] $M_1 = M_1 = 2$	This model $M_1 = M_1 = 2$
0.250	-	-	-	5.875-j 1.830	5.849-j 1.861
0.255	-	-	-	6.412-j1.820	6.381-j 1.850
0.260	-	-	-	6.799-j1.803	6.766-j 1.831
0.270	7.020-j 1.491	-	7.372-j 1.780	7.375-j 1.780	7.341-j 1.802
0.300	8.225-j 1.596	8.568-j 1.962	8.515-j 1.857	8.519-j 1.859	8.483-j 1.862
0.500	9.129-j 3.675	9.488-j 4.022	9.472-j 3.988	9.474-j 3.993	9.496-j 3.990
1.000	8.889-j 3.360	9.229-j 3.706	9.211-j 3.664	9.213-j 3.668	9.222-j 3.667
2.000	8.847-j 3.290	-	9.164-j 3.596	9.166-j 3.596	9.171-j 3.595
4.000	8.837-j 3.271	-	9.153-j 3.578	9.155-j 3.578	9.160-j 3.577

B: $f = 600\,\text{MHz}$, Moist earth: $\varepsilon_{r1} = 10$, $\sigma_1 = 10\,\text{mS/m}$, $n_{10} = 3.16 - j0.05$.

h/λ_0	Chang-Wait [7]	Božilović [8]	SIE-TP [2] $M_1 = M_1 = 2$	SIE-H [2] $M_1 = M_1 = 2$	This model $M_1 = M_1 = 2$
0.250	-	-	6.715-j 1.771	6.714-j 1.764	6.672-j 2.105
0.255	-	-	7.250-j 1.810	7.248-j 1.806	7.137-j 2.107
0.260	-	-	7.627-j 1.850	7.625-j 1.846	7.468-j 2.111
0.270	7.636-j 1.781	7.995-j 2.082	8.165-j 1.943	8.163-j 1.940	7.952-j 2.135
0.300	8.592-j 2.034	8.900-j 2.311	9.087-j 2.311	9.085-j 2.307	8.839-j 2.317
0.500	8.982-j 3.514	9.312-j 3.837	9.247-j 3.820	9.245-j 3.817	9.345-j 3.844
1.000	8.863-j 3.321	9.182-j 3.627	9.170-j 3.617	9.168-j 3.614	9.193-j 3.630
2.000	8.842-j 3.280	9.157-j 3.585	9.156-j 3.582	9.155-j 3.578	9.165-j 3.585
4.000	8.837-j 3.271	9.152-j 3.575	9.153-j 3.574	9.151-j 3.571	9.158-j 3.574

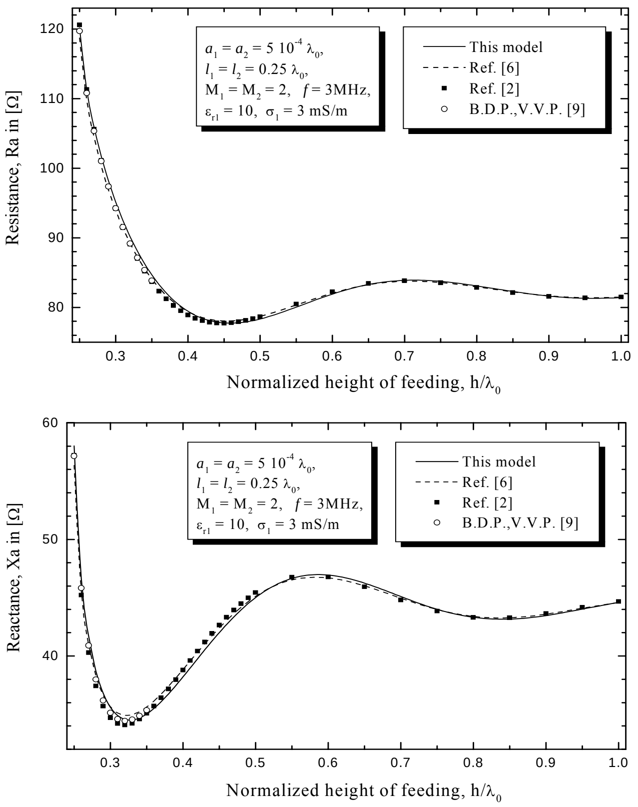

Figure 2: Input resistance and reactance of the half-wave dipole placed above a homogenous and isotropic medium as a function of the height of its centre h, $h \geq l_2$. Comparing of the results of different authors.

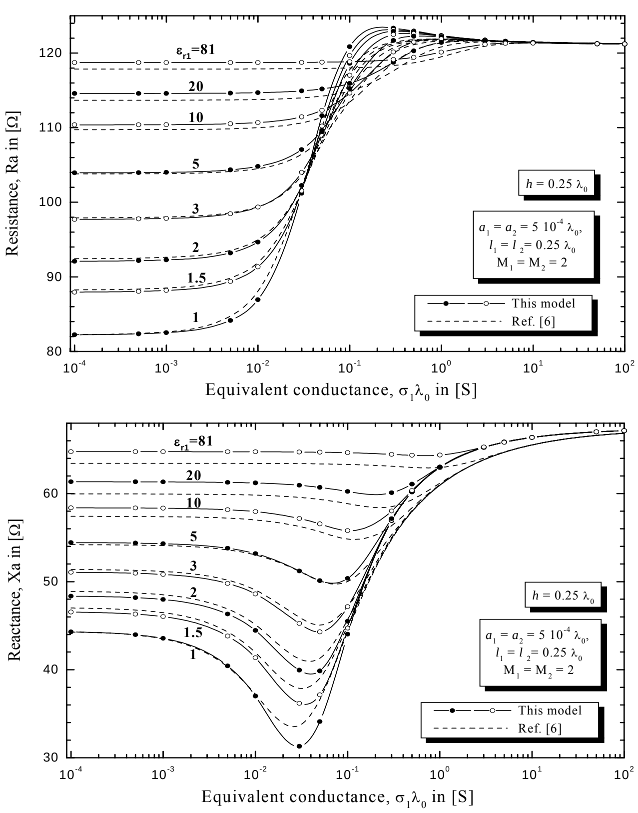

Figure 3: Input resistance and reactance of the half-wave dipole placed above a homogenous and isotropic medium as a function of the equivalent conductance, for different values of the permittivity as a parameter. Comparing of two models.

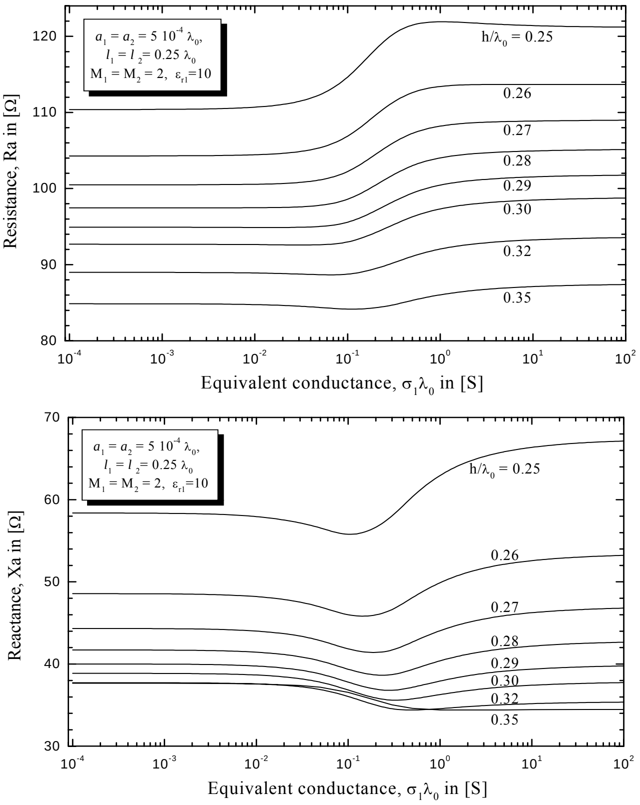

Figure 4: Input resistance and reactance of the half-wave dipole placed above a homogenous and isotropic medium as a function of the equivalent conductance and for different values of the normalized height of feeding as a parameter.

IV CONCLUSION

An improved model for analyzing vertical dipole antennas placed above homogenous and isotropic medium, is presented in this paper. Considering its simplicity, this model is similar to the previously proposed ones ([2], [3], [4] and [5]).

From the presented numerical results, one can conclude:

- The proposed model and the one in [2], give the results for the input impedance that are in very good accordance with the ones of the other authors under the condition $n_{10} \gg 1$, i.e. if $n_{10} > 3$;
- For the values of the refraction index $1 \le n_{10} < 3$, the proposed model and the one in [6] give the expected results for the input impedance, e.g. for $\varepsilon_{r1} \to 1$ and $\sigma_1\lambda_0 < 1\,\text{mS}$ the obtained results for the input impedances characterize the VDA in the free space. This is not the case with the models in [2]-[5].

This model has a general character, since it is not limited by the values of the refraction index, which is not the case with the previously proposed ones. The model is similar to the one in [6] with a difference that the constant A is deduced from the condition for the Brewster's angle ([12]).

REFEFENCES

[1] Sommerfeld, Partial Differential Equations in Physics, Academic Press Inc., Publishers, New York, 1949.

[2] Rančić P.D., Kitanović M.I., *A New Model for Analysis of Vertical Asymmetrical Linear Antenna Above a Lossy Half-Space*, Int. J. Electron. Commun. AEÜ, Vol.51, No. 3, pp. 155-162, 1997.

[3] Rančić P.D., Surutka J.V., Kitanović M.I., *The Influence of Finite Ground Conductivity on Characteristics of a Vertical Mast Antenna With Elevated Feeding*, Proc. of the II Int. Symp. EMC'96, pp. L1-2/427-432, Roma, Sep. 1996. (Poster Presentation Best Paper Award)

[4] Rančić P.D., Stevanović Z.D., Surutka J.V., *Coupled Horizontal Dipole Antennas Above a Conducting Half-Space*, Proc. of the III Int. Symp. EMC'98, pp. Q1-12/725-730, Roma, Sep. 1998.

[5] Rančić P.D., *An Efficient Method for Analysis of Horizontal Wire Antennas Placed Above a Ground Plane*, Proc. of the IV Int. Symp. EMC'2000, Vol.1, pp. 53-58, Brugge, Sep. 2000.

[6] Rančić M.P., Rančić P.D., *Vertical Linear Antennas in the Presence of a Lossy Half-Space: An Improved Approximative Model*, Int. J. Electron. Commun. (AEÜ), 2004. (offered for review and publication).

[7] Chang D.C., Wait J.R., *Theory of a Vertical Tubular Antennas Located above a Conducting Half-space*, IEEE Trans. on A.P., Vol. AP-18, pp. 182-188, 1970.

[8] Božilović H.A., *Vertical Symmetrical Cylindrical Dipole Antenna Located above a Conducting Half-space*, Int. J. Electron. Commun. (AEÜ) 46, pp. 361-364, 1992.

[9] Popović B.D., Petrović V.V., *Vertical Wire Antenna above Ground: Simple Near-exact Image Solution*, IEE Proc. Pt. H 140, pp. 501-507, 1993.

[10] Harrington R.F., *Field Computation by Moment Methods*, Mc Millan, New York, 1968.

[11] Popović B.D., Dragović M.B., Djordjević A.R., *Analysis and Synthesis of Wire Antennas*, RSP, John Willey & Sons LTD, Chichester, England, 1982.

[12] Mahmoud S.F., Metwally A.D., *New Image Representation for Dipoles near a Dissipative Earth 1. Discrete Images 2. Discrete Plus Continuous Images*, Radio Sci., Vol. 16, pp. 1271-1283, 1981.

Acknowledgement: The first author thanks dr Hartmut Brauer and the DAAD foundation for the support while preparing the paper.

Sadržaj: U ovom radu određena je nepoznata raspodela struje i ulazna impedansa/admitansa vertikalne dipol antene u prisustvu homogene i izotropne poluprovodne sredine. Metod podešavanja u tačkama i celodomenska polinomska aproksimacija za nepoznatu raspodelu struje su korišćeni za numeričko rešavanje sistema integralnih jednačina Hallen-ovog tipa (SIE-H). Uticaj konačne provodnosti zemlje, iskazan Sommerfeld-ovim integralnim jezgrom (SIK), modelovan je na nov jednostavan način i bez ograničenja za vrednost indeksa refrakcije. Ovaj aproksimativni model, pored jednostavnosti i opštosti, dao je izlazne numeričke rezultate zavidne tačnosti.

JEDAN APROKSIMATIVNI MODEL ZA ANALIZU VERTIKALNIH ANTENA IZNAD ZEMJE

Rančić M., Rančić P.