COUPLED VERTICAL DIPOLES ABOVE GROUND: SELF- AND MUTUAL- ADMITTANCES/IMPEDANCES

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I INTRODUCTION

A simple model for numerical evaluation of Sommerfeld's integrals (Sommerfeld's Integral Kernel - SIK), which occur when analyzing vertical conductors placed above a lossy half-space, was proposed at the last-year TELFOR Conference [1]. Real inhomogeneous ground was considered a homogenous, isotropic medium of known electrical parameters (σ_1 , $\varepsilon_1 = \varepsilon_{r1}\varepsilon_0$, $\mu_1 = \mu_0$). The proposed model for the SIK is one of many simple ones that were developed at the Faculty of Electronic Engineering in Niš for analysis of the influence of the finite ground conductivity on the characteristics of wire structures ([2] - [4]). The validity of the proposed model was tested on the example of the vertical asymmetrical dipole placed above a lossy half-space, comparing the results for the input admittance/impedance with the corresponding ones of other authors ([5] - [7]). Based on numerical experiments and the presented output results for the input admittance/impedance and the unknown current distribution, and their good accordance with the results from the references, it was concluded that the proposed model for the SIK, in the surroundings of the antenna, has a simple form, general character (it does not depend on the values of the electrical parameters of the lossy half-space $\varepsilon_{r1} \in [1,81]$, $\sigma_1\lambda_0 \in [0,\infty]$) and enviable accuracy.

In this paper, the SIK model proposed in [1] was used for modeling the expressions for the Hertz's vector and electrical scalar potentials of the system of coupled vertical dipoles placed above homogenous and isotropic medium. Then, the point-matching method for numerical solving of the system of integral equations of Hallen's type (SIE-H) was used for determining the unknown current distributions - UCDs, which were considered in the polynomial form ([10], [11]).

The coupled vertical dipoles were observed in the so-called symmetrical and asymmetrical working regimes, which allows us to determine self- and mutual-admittances/impedances and the corresponding current distributions. These results also enable determining of the UCDs and input admittance/impedance of a system consisting of a vertical dipole and a parasite radiator placed above a homogenous and isotropic lossy half-space.

Among numerous numerical results, a number of those that prove the validity of the proposed model will be shown, as well as a number of those for self- and mutual-admittances/impedances of the coupled dipoles, and for input impedances of the vertical dipole with a parasite element.

II THEORETICAL BACKGROUND

II.1 DESCRIPTION OF THE MODEL GEOMETRY

A system of two vertical dipole antennas of conductor length l_k , $k = 1,2,3,4$, and circular cross-section radius a_k , $a_k << l_k$ and $a_k << \lambda_0$ (λ_0 - wave-length in air), arbitrary placed in the air above a homogenous and isotropic lossy half-space, is considered. Electrical parameters of the air $\sigma_0 = 0$, ε_0 and μ_0 and the semi-conducting medium σ_1 , $\varepsilon_1 = \varepsilon_{r1}\varepsilon_0$ and $\mu_1 = \mu_0$ (σ_i - conductivity, ε_i - permittivity and $\mu_i = \mu_0$ - permeability, $i = 0,1$), are known. Following labels are also introduced: $\underline{\sigma}_i = \sigma_i + j\omega\varepsilon_i = j\omega\varepsilon_0\underline{\varepsilon}_{ri}$, $\underline{\varepsilon}_{ri} = \varepsilon_{ri} - j60\sigma_i\lambda_0$, $i = 0,1$ - complex conductivity and complex relative permittivity, respectively; $\underline{\gamma}_i = \alpha_i + j\beta_i = (j\omega\mu_i\underline{\sigma}_i)^{1/2}$, $i = 0,1$ - complex propagation constant; $\underline{n}_{10} = \underline{\gamma}_1 / \underline{\gamma}_0$ - refraction coefficient; and $\omega = 2\pi f$ - angular frequency.

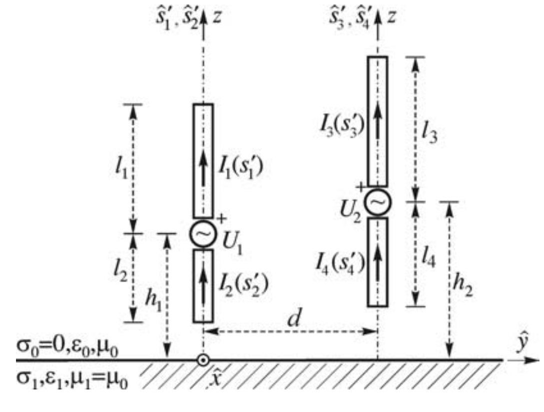

Figure 1: Schematic illustration of two coupled asymmetrical vertical dipole antennas placed above a lossy half-space.

The UCDs along the antenna conductors are localized along the axes and denoted by $I_k(s'_k)$, $0 \le s'_k \le l_k$, $z'_k = z_{Ak} + s'_k$, $k = 1,2,3,4$. The antennas are fed by two ideal voltage generators of voltages U_1 and U_2 , and frequency f . Schematic illustration of the antenna system is given in **Figure 1**.

II.2 HERTZ'S VECTOR AND ELECTRICAL SCALAR POTENTIAL

In the surroundings of the conductor, the Hertz's vector potential only has z - component, $\vec{\Pi}_0 = \Pi_{z0}\hat{z}$, expressed by

$$\Pi_{z0} = \frac{1}{4\pi\underline{\sigma}_0} \sum_{k=1}^{4} \int_{s'_k=0}^{l_k} I_k(s'_k)[K_0(r_{1k}) + S_{00}^{v}(r_{2k})]\mathrm{d}s'_k \ , \quad (1)$$

where: $K_0(r_{ik}) = \exp(-\underline{\gamma}_0 r_{ik}) / r_{ik}$, $i = 1,2$, $k = 1,2,3,4$ is the standard potential kernel in the air; r_{ik} - distances from the source, $i = 1$, and the image, $i = 2$, to the observed field point; and $S_{00}^{v}(r_{2k})$ - the Sommerfeld's integral kernel (SIK). If we apply the approximation proposed in [1], the SIK gaines the following approximated form

$$S_{00}^{v}(r_{2k}) \cong B[K_0(r_{2k}) + (\underline{\gamma}_0 / C) L(r_{2k})], \qquad (2)$$

where $B = (\underline{n}_{10}^2 - 1)/(\underline{n}_{10}^2 + 1)$ and $C = -(\underline{n}_{10}^2 + 1)^{1/2}$ are constants obtained as in [1], and $L(r_{2k})$ - new integral kernel:

$$L(r_{2k}) = -\int_{v=0}^{z+z_k'} K_0(r_{2kv})\,\mathrm{d}v - \frac{\pi}{2}[N_0(\beta_0\rho_k') + j\,J_0(\beta_0\rho_k')] ,(3)$$

where ρ_k' is radial distance, $\rho_k' = [(x-x_k')^2 + (y-y_k')^2]^{1/2}$, and $N_0(\beta_0\rho_k')$ and $J_0(\beta_0\rho_k')$ are the zero-th Neuman and Bessel functions of the first kind, respectively.

Using this approximation, Eq. (1) becomes

$$\Pi_{z0}(\vec{r}) = \frac{1}{4\pi\underline{\sigma}_0} \sum_{k=1}^{4} \int_{s_k'=0}^{l_k} I_k(s_k')[K_0(r_{1k}) + BK_0(r_{2k}) +$$

$$+\underline{\gamma}_0 (B/C) L(r_{2k})]\,\mathrm{d}s_k' . \qquad (4)$$

Electrical scalar potential, $\varphi_0 = -\mathrm{div}\,\vec{\Pi}_0 = -\partial\Pi_{z0}/\partial z$, is

$$\varphi_0(\vec{r}) = \frac{-1}{4\pi\underline{\sigma}_0} \sum_{k=1}^{4} \int_{s_k'=0}^{l_k} I_k(s_k') \left\{ \frac{\partial}{\partial s_k'} \left[-K_0(r_{1k}) + B\,K_0(r_{2k}) \right] - \right.$$

$$\left. -\underline{\gamma}_0 (B/C) K_0(r_{2k}) \right\} \mathrm{d}s_k' . \qquad (5)$$

II.3 SYSTEM OF INTEGRAL EQUATIONS OF HALLEN'S TYPE

Canonical form of the system of integral equations of Hallen's type used in this paper for evaluation of the UCDs is, according to [3], in the following form

$$\Pi_{z0}(s_n) = +\frac{1}{4\pi\underline{\sigma}_0} \pi_n \,\mathrm{ch}(\underline{\gamma}_0 s_n) - \frac{V_{en}}{\underline{\gamma}_0} \mathrm{sh}(\underline{\gamma}_0 s_n) \qquad (6a)$$

$$\varphi_0(s_n) = -\frac{\underline{\gamma}_0}{4\pi\underline{\sigma}_0} \pi_n \,\mathrm{sh}\,(\underline{\gamma}_0 s_n) + V_{en} \,\mathrm{ch}\,(\underline{\gamma}_0 s_n) \qquad (6b)$$

where: π_n and V_{en} - unknown integration constants, and s_n - local coordinate along the generatrice of the n-th conductor, $0 \le s_n \le l_n$, $n = 1,2,3,4$. $\Pi_{z0}(s_n)$ and $\varphi_0(s_n)$ are evaluated according to Eqs. (4) and (5).

The UCDs are assumed to be in a polynomial form with unknown current coefficients of degree M_k, which are determined, along with the unknown integration constants, using the point-matching method and satisfying certain conditions. Currents at free conductor ends equal zero and at connection points are continuous, and there is a potential leap at feeding points, i.e.:

$$\varphi_0(s_1 = 0^+) - \varphi_0(s_2 = h_1^-) = U_1 ,$$

$$\varphi_0(s_3 = 0^+) - \varphi_0(s_4 = h_2^-) = U_2 .$$

II.4 SELF- AND MUTUAL- ADMITTANCES/IMPEDANCES

The SIE-H given by Eqs. (6a, b) can be solved as described in the previous Section, but in the so-called symmetrical and asymmetrical working regimes. As a result of such analysis, the input admittances of the symmetrically and asymmetrically fed 1st and 2nd antenna, respectively, are obtained. Applying the theory of "Y" and "Z" parameters of the two-port network, self- and mutual- admittances/ impedances can be determined [8]. Once the "Y" and "Z" parameters are known, the input impedance of the antenna with a parasite element can be evaluated using the following expression:

$$Z_{ul} = R_a + \mathrm{j}X_a = (U_1/I_1)\|_{U_2=0} = Z_{11} - Z_{12}Z_{21}/Z_{22} . \ (7)$$

III NUMERICAL RESULTS

A software package for numerical calculations was formed based on the described procedure. Using this program package numerous numerical experiments were done, and some of the obtained results will be presented in this Section. The applied polynomial current approximation is of $M_1 = \cdots = M_4 = 2$ degree, in all of the examples.

Values of the self- and mutual- admittance of the coupled vertical symmetrical dipoles placed above a lossy half-space ($\sigma_1\lambda_0 = 10^{-5}$ S and $\varepsilon_{r1} = 1.03$) obtained using the new proposed model for the SIK, are given in **Table 1** as well as the corresponding results from [1], [4], [8] and [9]. Excellent agreement of the results is evident.

Table 1: Self- and mutual- admittances of the coupled vertical symmetrical dipoles placed above a lossy half-space: $a_1 = \cdots = a_4 = 0.007\lambda_0$, $l_1 = \cdots = l_4 = 0.25\lambda_0$, $d = 0.25\lambda_0$, $h_1 = 0.3\lambda_0$, $h_2 = 0.55\lambda_0$, $\sigma_1\lambda_0 = 10^{-5}$ S, $\varepsilon_{r1} = 1.03$.

	$Y_{11} = G_{11} + \mathrm{j}B_{11}$ [mS]	$Y_{12} = G_{12} + \mathrm{j}B_{12}$ [mS]
This model	7.527 - j 4.082	0.780 + j 3.989
Ref. [11]	7.519 - j 4.097	0.783 +j 3.995
Ref. [8]	7.514 - j 4.091	0.785 + j 3.991
Ref. [9]	7.440 - j 4.105	0.810 + j 3.958
Ref. [4]	7.526 - j 4.078	0.781 + j 3.988

The results for the self-admittance of the coupled vertical dipole antennas placed above a homogenous and isotropic medium, versus normalized height of feeding h/λ_0 ($h_1 = h_2 = h$) are shown in **Figure 2**. The distance between the antenna elements is $d = 0.1\lambda_0$, and the electrical parameters of the ground: $\sigma_1\lambda_0 = 0.3$S, $\varepsilon_{r1} = 10$.

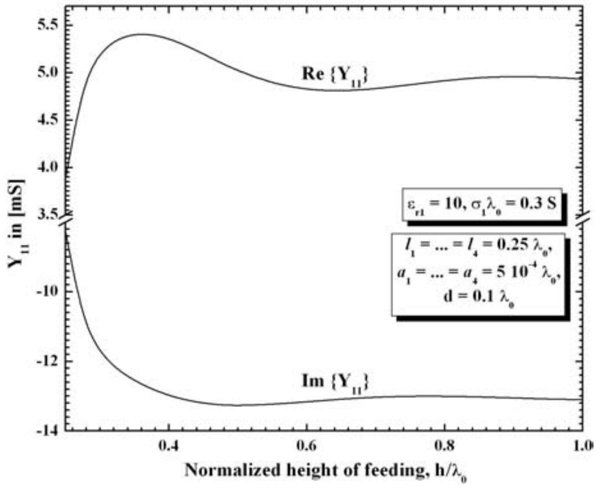

Figure 2: Self-admittance of the coupled vertical dipole antennas placed above a lossy half-space, versus normalized height of feeding h/λ_0, $h_1 = h_2 = h$.

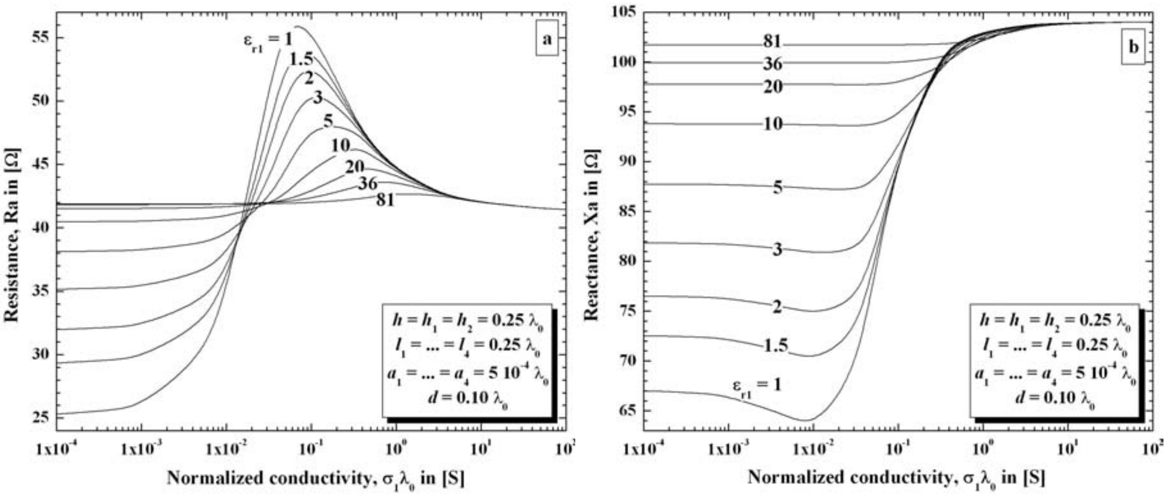

Figure 3: Input resistance and reactance of the vertical dipole antenna with a vertical parasite element, versus normalized conductivity $\sigma_1\lambda_0$, for various values of the relative permittivity ε_{r1} as a parameter.

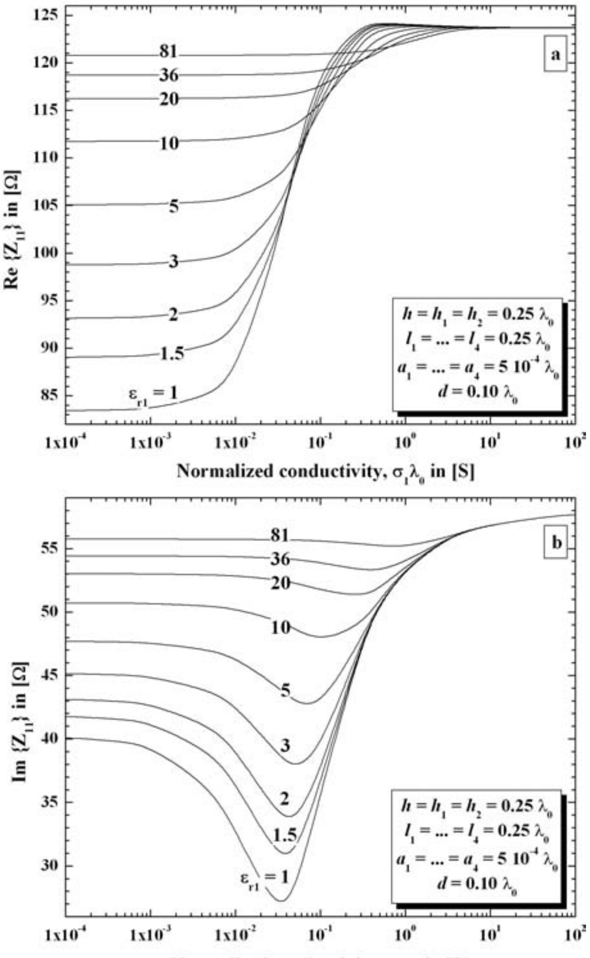

Figure 4: Self-impedance of the vertical coupled dipole antennas, versus normalized conductivity $\sigma_1\lambda_0$, for various values of the relative permittivity ε_{r1} as a parameter.

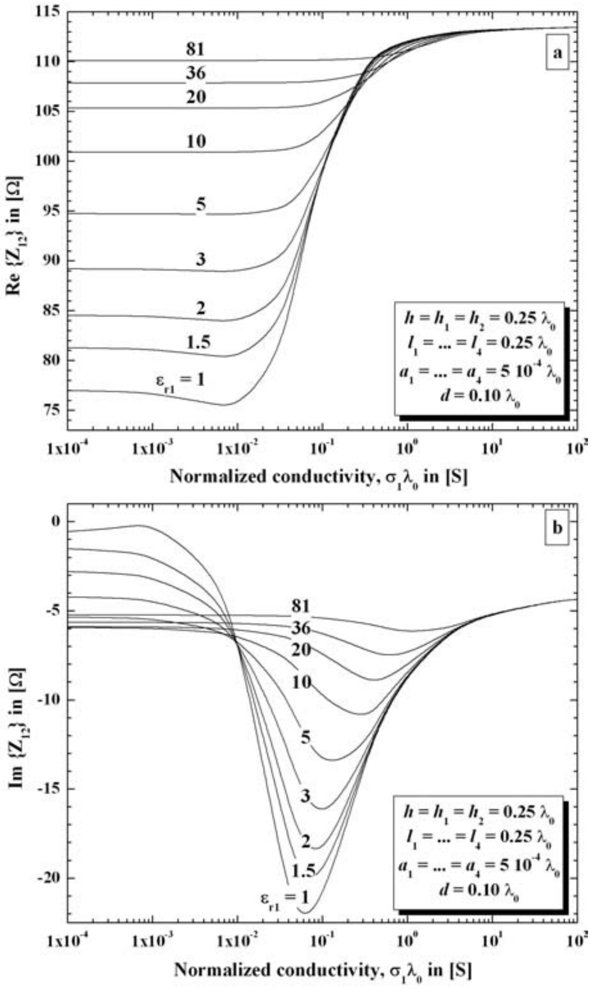

Figure 5: Mutual-impedance of the vertical coupled dipole antennas, versus normalized conductivity $\sigma_1\lambda_0$, for various values of the relative permittivity ε_{r1} as a parameter.

The results for the input resistance and reactance of the vertical dipole antenna with a vertical parasite element, as a function of normalized conductivity $\sigma_1\lambda_0$, and for various values of the relative permittivity ε_{r1} as a parameter, are shown in **Figs. 3a, b**. The distance between the antennas is $d = 0.10\,\lambda_0$. The results for the self- and mutual-impedances of two vertical coupled dipole antennas, versus normalized conductivity are shown in **Figs. 4a, b** and **5a, b**, respectively.

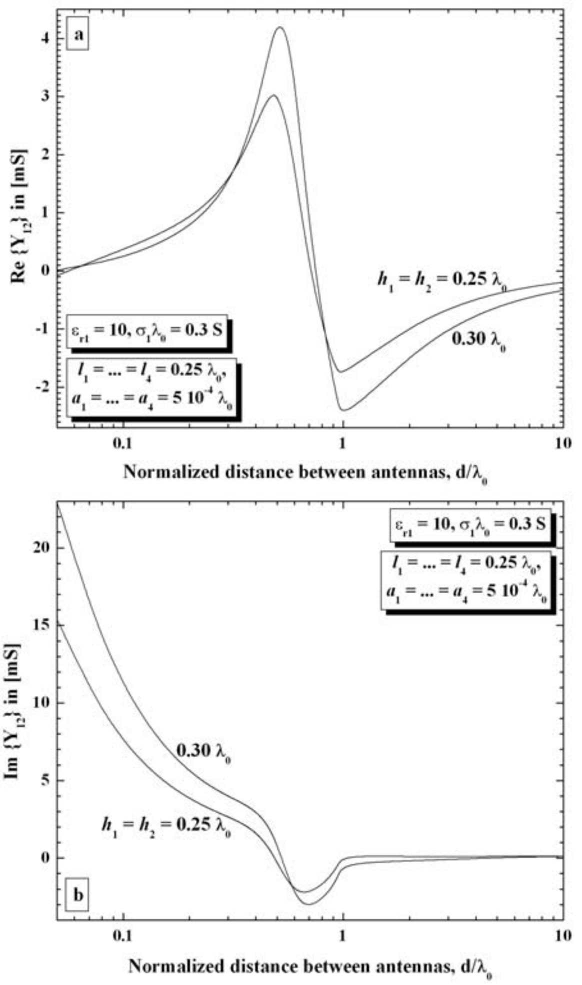

Figure 6: Mutual-admittance of the vertical coupled dipole antennas, versus normalized distance between antennas d/λ_0, for two different values of antenna feeding $h_1 = h_2$ as a parameter.

The results for the self- and mutual-admittances of the vertical coupled dipole antennas, versus normalized distance between antennas d/λ_0 are shown in **Figs. 6a, b**. The height of the antenna feeding is taken to be parameter ($h_1 = h_2 = 0.25\lambda_0$ and $0.30\lambda_0$).

IV CONCLUSION

A new model for analyzing vertical dipole antennas, proposed in [1], was used in this paper for solving a problem of two coupled asymmetrical vertical dipole antennas arbitrary placed above a lossy half-space that is treated as a homogeneous and isotropic medium. This new model has a general character, since it is not limited by the values of the electrical parameters of the ground; it has a simple form and gives highly accurate results.

The UCDs, which were assumed to be in a polynomial form, and the input impedance of the antenna, were determined numerically using the point-matching method while solving the SIE-H. A number of numerical experiments were performed and the obtained results were graphically illustrated.

This simple, general and accurate model was also used for solving a problem of a vertical dipole antenna with a vertical parasite element.

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Sadržaj: U ovom radu su određene nepoznate raspodele struja (UCDs) sistema dve spregnute vertikalne dipol antene proizvoljno postavljene iznad poluprovodne sredine. Nepoznate raspodele struja su određene numeričkim rešavanjem sistema integralnih jednačina Hallen-ovog tipa (SIE-H) korišćenjem metoda podešavanja u tačkama, pri čemu je za struje korišćena polinomska aproksimacija. U radu su takođe određene sopstvene i međusobne admitanse/impedanse sistema dve spregnute vertikalne dipol antene, kao i ulazna impedansa vertikalnog dipola sa vertikalim parazitnim elementom. Uticaj poluprovodne sredine iskazan Sommerfeld-ovim integralnim jezgrom (SIK) modelovan je na jednostavan, opšti i vrlo tačan način.

SPREGNUTI VERTIKALNI DIPOLI IZNAD ZEMLJE: SOPSTVENE I MEĐUSOBNE ADMITANSE/ IMPEDANSE

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