PERFORMANCE ANALYSIS OF \( \tau \)-CDMA SYSTEM WITH COHERENT DETECTION

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I INTRODUCTION

Two the most often used versions of spread spectrum systems are frequency hopping (FH) and direct sequence (DS) configurations. Due to simplicity, DS configuration has been accepted for civil applications in mobile communication systems (ex. Standards IS-95, IS-665, W-CDMA UMTS). For large bit rates DS system will have low processing gain and performance of a Rake receiver will be considerably degraded. In the presence of near-far effect (imperfect power control) DS systems should use multiuser detectors (optimal or sub optimal structures). Since the optimum DS receiver is difficult to implement, the communicator may prefer to use a frequency-hopping spread-spectrum (FH-SS) system instead. For these reason these systems are used in both military and civil applications. For example the advanced versions of TDMA (GSM) land mobile communications systems are using also hopping to improve performance in fading channel and reduce intercell interference.

In order to improve performance in jamming and fading environment these systems can use different techniques. Traditionally, diversity was obtained via multiple hops per information (or coded) symbol. Such a fast hopping makes difficult the synchronization of the carrier phase and, consequently, imposes the use of a noncoherent receiver. Thus, a significant loss in error performance results, due to both noncoherent demodulation and noncoherent combining of the received diversity replicas. Taking into account these losses, and using binary frequency-shift keying (BFSK) modulation, an optimum diversity scheme is analyzed in [1] for the worst case jammer and with side information on noise and jamming levels. Since optimum diversity has more analytical value than practical existence, the error probability is much higher in practice. In order to recover these performance losses, some authors have studied a solution that makes coherent reception feasible; see, e.g., [2], [3], [4].

Frequency diversity as used on Rayleigh fading channels, and which differs from the diversity mentioned above, was proposed in [5] to counter band-limited interference. Such a diversity allows one to avoid noncoherent combining loss. In this system, called frequency-diversity spread spectrum (FD-SS), the communicator frequency band is partitioned into \( N \) disjoint subbands on which \( N \) replicas of the signal are simultaneously transmitted. However, since frequency hopping is considered as mandatory in some applications, some solutions combine both FD-SS and FH-SS systems [1]. The main objective is to guarantee coherent demodulation and to avoid noncoherent combining losses.

In this paper we introduce a new concept of multiple access called \( \tau \)-CDMA. This system combines good characteristics of DSSS and FHSS systems. Near far effect is mitigated without need for complicated multiuser detectors and at the same time simplicity of DSSS system is preserved. No need for frequency synthesizer and coherency for coherent Rake receive is maintained in much simpler way than in the FH system. The concept is based on a modification of Direct Sequence Spread Spectrum (DSSS) system where transmitted waveform includes multiple amplitude and delay replicas of DSSS signal. The notation \( mD-DSDH \) will be used for DS signal that includes \( m \) delayed replicas sent in a limited delay window of \( M \) chip intervals. The position of the delay window is hopped (Delay Hopping) in the range of the code length \( N \). This should provide resistance to near far effect. If the signal energy is split to \( m > 1 \) separate components making it more vulnerable to noise and fading, the overall flow of useful information will be still increased. The results demonstrate that under the large range of the signal, channel and interference parameters this system offers better performance.

II SYSTEM MODEL

For a standard CDMA concept the simplest form of the overall received signal can be represented as

\[
r(t) = \sum b_k(t) s_k(t - \tau_k) \cos(\omega t + \theta_k) + i(t) + n(t)
\]

where for the \( k \)-th user \( b_k \) and \( s_k \) are data (bits) and PN sequence respectively, \( i(t) \) is the total interference, and \( n(t) \) is Gaussian noise. The standard receiver uses coherent despreading and demodulation and will be referred to as coherent CDMA (c-CDMA). Extension to include I and Q signal component is straightforward. If a correlator (composed of a multiply plus integrate) is used for signal despreading then we will refer to this structure as correlator receiver (CR). If a PN matched filter is used at the receiver and if the sequence period \( T_c = N T_s \) equals bit period \( T_s \), then at the output of the filter, one correlation pulse generated by the useful signal will appear per bit interval. The correlation pulse will appear each time at the chip interval when the input sequence coincides with the filter coefficients. This will be referred to as PN matched filter receiver (PNMFR).

If now instead of sequence \( s_k \) a delayed version (cyclic shift) of the same sequence is used \( s'_k \) the position of the correlation pulse will depend on the sequence shift \( \tau = \mu_k(k) = kT_c \).

Eq.(1) now becomes

\[
r(t) = \sum b_k(t) s_k(t - \mu_k(k)T_s - \tau_k) \cos(\omega t + \theta_k)
\]

If \( \mu_k(k) = kT_c, k = 0,1,..,M - 1 \), is one out of \( M = 2^n \) different adjacent cyclic shifts then \( n = \log_2 M \) additional bits can be transmitted within one symbol interval. Capacity of a
standard coherent CDMA system is roughly \[ K \equiv G / y_b \] (3)
where \( G \) is the system processing gain and \( y_b \) is signal to noise ratio needed for a given quality of transmission. In our case \( G = N \). In accordance with the above explanation capacity of the new CDMA system is additionally increased by a factor
\[
K' \cong (G \log_2 M) / y'_b = (N \log_2 M) / y'_b
\] (4)
where \( y'_b \) is signal to noise ratio needed for the same BER. Parameter \( y'_b \) will depend on the type of demodulator and is the main subject of this paper.

Let's suppose that now instead of sending one out of \( M \) delayed versions of the signal we send two different delayed replicas simultaneously. If amplitudes are the same we can form
\[
M_2 = \sum_{r=1}^{M-1} M(M-1) / 2
\] (5)
different combinations and send \( n_2 = 2 \log(M - 1) / 2 = \log M + \log(M - 1) - 1 \) bits. If \( M \) is large \( n_2 \approx 2 \log M - 1 \approx 2n \) is almost twice as much as in the case of simple \( M \)-ary modulation. The optimum receiver will now have to find two the largest samples at the output of the matched filter. If now instead of two, \( m \) out of \( M \) signal replicas are simultaneously transmitted we have \( mM \tau \)-DSSS modulation and corresponding \( mM \tau \)-CDMA system. The number of transmitted bits per \( T_s \) is now further increased. A simple calculus is needed to evaluate capacity improvement in noise free channel. Receiver block diagram is shown in Fig.1, where \( u_{in}^* = s(t - kT_c - \tau) \).

The interference \( i(t) \) is modelled as a Gaussian interference with zero mean and variance
\[
\sigma_i^2 \equiv (K-1)m \alpha^2 \bar{A}^2 / G
\] (6)
where \( \alpha \) is the channel attenuation and \( \bar{A}^2 \) is the average amplitude of the interferers.

### III PERFORMANCE ANALYSIS
As performance measure we will be discussing symbol error rate or the system efficiency improvement factor defined as [7]
\[
E = \left[ (1 - P) / P_0 \right] / \left[ (1 - P_0) / P_0 \right]
\] (7)
where parameters with index zero refer to the standard modulation, \( n \) is the number of bits per symbol, \( P \) is the bit error rate and \( (1 - P) / P_0 \) is the average number of correctly transmitted bits per symbol.

Signals at the output of integrators are Gaussian distributed, with the following pdfs.
\[
p(U_k) = \frac{1}{\sqrt{2\pi(\sigma^2_k + \sigma^2_\alpha)}} \exp \left[ -\frac{(U_k - a_k)^2}{2(\sigma^2_k + \sigma^2_\alpha)} \right]
\] (8)
\[
p(U_k) = \frac{1}{\sqrt{2\pi(\sigma^2_k + \sigma^2_j)}} \exp \left[ -\frac{U_k^2}{2(\sigma^2_k + \sigma^2_j)} \right]
\] (9)
where \( a_k = \alpha A \) is the received useful signal amplitude, \( \sigma^2_\alpha \) is the additive Gaussian noise variance, and \( \sigma^2_j \) is the interference power.

Probability of a correct decision is
\[
P_e = P(U_1 > U_{m+1}, U_1 > U_{m+2}, \ldots, U_1 > U_M) \times
P(U_2 > U_{m+1}, U_2 > U_{m+2}, \ldots, U_2 > U_M) \times \ldots \times P(U_m > U_{m+1}, U_m > U_{m+2}, \ldots, U_m > U_M)
\] (10)
where
\[
P(U_1 > U_{m+1}, U_1 > U_{m+2}, \ldots, U_I > U_M) =
\int_{-\infty}^{\infty} P(U_1 > U_{m+1}, U_1 > U_{m+2}, \ldots, U_I > U_M \mid U_1) p(U_1) dU_1
\] (11)
\[
P(U_2 > U_{m+1}, U_2 > U_{m+2}, \ldots, U_2 > U_M) =
\int_{-\infty}^{\infty} P(U_2 > U_{m+1}, U_2 > U_{m+2}, \ldots, U_2 > U_M \mid U_2) p(U_2) dU_2
\]
\[
\vdots
\]
\[ P(U_m > U_{m+1}, U_m > U_{m+2}, ..., U_m > U_M) = \]
\[ = \int_{-\infty}^{\infty} P(U_m > U_{m+1}, ..., U_m > U_M | U_m) p(U_m) dU_m \]  
\[ = \frac{1}{2} \left( 1 + \text{erf} \left( \frac{U_i}{\sqrt{2(\sigma_i^2 + \sigma_j^2)}} \right) \right) \]  
\[ P(U_1 > U_{m+1}, U_1 > U_{m+2}, ..., U_1 > U_M) = \]
\[ = \int_{-\infty}^{\infty} P(U_1 > U_{m+1} | U_1) p(U_1) dU_1 \]  
\[ = \frac{1}{2^{M-m}} \sqrt{\frac{\pi}{2}} \int_{-\infty}^{\infty} e^{-x^2} \left[ 1 + \text{erf} \left( x + \frac{a_i^2}{2(\sigma_i^2 + \sigma_j^2)} \right) \right] dx \]

Probability of a correct decision is
\[ P_c = \left( \frac{1}{2^{M-m}} \sqrt{\frac{\pi}{2}} \int_{-\infty}^{\infty} e^{-x^2} \left[ 1 + \text{erf} \left( x + \frac{a_i^2}{2(\sigma_i^2 + \sigma_j^2)} \right) \right] dx \right)^m \]

The error probability now becomes
\[ P_e = 1 - P_c = F_{\text{coh}}(a_i^2) \]
\[ = 1 - \left( \frac{1}{2^{M-m}} \sqrt{\frac{\pi}{2}} \int_{-\infty}^{\infty} e^{-x^2} \left[ 1 + \text{erf} \left( x + \frac{a_i^2}{2(\sigma_i^2 + \sigma_j^2)} \right) \right] dx \right)^m \]

Due to Rayleigh fading, \( a_i \) has the following pdf:
\[ f(a_i) = \frac{a_i}{\sigma_i^2} \exp \left( -\frac{a_i^2}{2\sigma_i^2} \right) \]

where \( 2\sigma_i^2 \) is the signal power.
After averaging over \( a_i \), the error probability becomes
\[ P_e = \int_{0}^{\infty} F_{\text{coh}}(x \cdot 2\sigma_i^2) \exp(-x)dx \]  

**IV NUMERICAL RESULTS**

In this section we present some examples of numerical results in order to illustrate advantages of using \( \tau \)-CDMA system. Efficiency improvements as a function of number of users for coherent detection and no fading channel is presented in Fig.2. The set of curves is obtained for two values of SNR (4 and 16 dB) and a number of combinations for \( M \) and \( m \). For high SNR (16 dB) parameter \( E \) is larger than one for the large scale of parameters. For smaller \( K \), \( E \) is higher if \( m \) is larger. If \( m \) is lower the maximum value of \( E \) is lower but \( E \) remains above one for higher \( K \).
Fig. 3 represents the same results as Fig. 2 but in the case when the signal is propagating through Rayleigh fading channel. In general, improvement factor is lower and only for the few users in the channel larger than one.

V CONCLUSION
In this paper we introduce a new concept of multiple access called $\tau$-CDMA. The system provides resistance to near far effect without need for complex multiuser detectors. If the signal energy is split to $m > 1$ separate components, making it more vulnerable to noise and fading, the overall flow of useful information will be still increased. The results demonstrate that under the large range of the signal, channel and interference parameters this system offers better performance. This system combines good characteristics of DSSS and FHSS systems.

REFERENCES

Abstract: In this paper we introduce a new concept of multiple access called $\tau$-CDMA. The abbreviation stands for Code Division Multiple Access with delay ($\tau$) modulation and hopping. The concept is based on a modification of Direct Sequence Spread Spectrum (DSSS) system where transmitted waveform includes multiple amplitude and delay replicas of DSSS signal. In general notation $mM\tau$-DSDH will be used for DS signal that includes $m$ delayed ($\tau$) replicas sent in a limited delay window of $M$ chip intervals. If the signal energy is split to $m > 1$ separate components, making it more vulnerable to noise and fading, the overall flow of useful information will be still increased. The results demonstrate that under the large range of the signal, channel and interference parameters this system offers better performance.

ANALIZA PERFORMANSI $\tau$-CDMA SISTEMA SA KOHERENTNOM DETEKCIJOM, Nenad Milošević, Bojan Dimitrijević, Milan Živković, Aleksandra Cvetković, Zorica Nikolić