Geometrical equivalents of ad hoc cell clusters

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Abstract: In this paper we proposed equivalents of ad hoc network at two structural levels. At first, we gave a geometrical equivalent of first-order cell aggregate, having the central cell surrounded by densely packed and disposed outer cells. Such compact dispositions are obviously extremal cases of aggregate appearance but make the modelling much easier, and give us a referent indication for real dispositions. For much complex ad hoc networks, consisted of many partially overlapped aggregates, where each of them possess internal switching and routing capabilities, we proposed vector-chain description that expresses routing aspects while keeping basic geometrical structure of the network.

Key words: Ad hoc network, cell aggregate, elliptic equivalent, vector-chain equivalent of multihop ad hoc networks.

I. INTRODUCTION

From the simplest sporadic rudiments ad hoc networks are advancing to complicated multihop and multilayered structures. Increasing node processing capabilities enable routing functions and formation of extended local area networks (ELAN).

Such networks exhibit versatile geometry of cell disposition as opposed to static cell structure in mobile networks or even stationary cell structure of satellite mobile networks. Geometrical variability stems from different features of network nodes, i.e., various mobile devices (cellular phones, PDA, intelligent sensor devices, earphones etc.) and their highly unpredictable behaviour [1]–[3].

We will assume that the basic geometrical element of any ad hoc network is a circle which radius is determined by device emission RF power. In reality, such radiation pattern represents simplified picture but it will suffice for our analysis. This parameter may be introduced as stochastic if we assume regulation of emission power in each mobile device within an ad hoc network depending on their distance in order to save the energy [4].

Ad hoc networks may take various levels of complexity. The simplest ones comprise from two to several locally connected mobile devices which form a single cell. Two or several of such first-order networks, connected over its peripheral mobile nodes form a second-order ad hoc network. Both of these types can be designated as truly or homogenous ad hoc networks.

One must notice relativity with which these two types of ad hoc networks are categorized. Even a first-order cell may be transferred into a second-order one if the nodes change their positions from a clustered to elongated disposition when the stations cannot communicate directly but over other ones serving as relays. But here, in this paper, we will neglect such possibility in order to functionally distinguish central node as main switch, correspondent to base station of mobile cell, and peripheral node which serves as router toward neighbouring cells.

To another category belong heterogeneous ad hoc networks where two to several first-order cells are locally connected via stationary communication facilities. Of course, this category can be extended to wide area distances but we will restrict ourselves only to local areas.

From the point of view of second-order network sometimes it would be significant to treat first-order network, i.e., cell cluster, as geometrical equivalent. It means all emission areas around mobile stations that form a cluster of overlapping circles to be represented by appropriate geometrical figure which describes them as the best replacement or equivalent. In this paper we propose how to find out such equivalents.

II. CLUSTER OF NODE EMISSION AREAS

Geometrically speaking, connection between two mobile devices begins when their mutual distance is below emission range of device with lower emission power (Fig.1.a). For several stations which make centralized aggregate geometrical representation could be very entangled. To make it simpler let us depict the disposition of node circular areas not with overlapping circles but with touching ones (Fig.1.a), having in mind that the central circle extends right over the centre of the biggest surrounding circle (Fig.1.b).

Now we can introduce a geometrical formalism where the circular emission range of a mobile device is the lowest structural unit in this model. First-order aggregate or assembly of such circles, but with various sizes, can be considered as an elementary higher-order structural unit. To define it some simplifications should be made.

- Instead of more natural 3D model we consider 2D model. It is more effective in simulations and gives more obvious interpretations. 3D ad hoc networks are especially prevailing in urban multifloor environments.
- The form of cell is circular – simplifies formulation of geometric relationships within aggregate.
- The cell size is taken as random, due to stochastic factors acted during establishing of ad hoc cell, since we assumed adjusting of emission power depending of mobile nodes distances.

Sizes of circular cells follow some discrete probability distribution

\[ R = \left\{ r_1, r_2, \ldots, r_N \right\} \]

or

\[ \left\{ R \right\} = \left\{ p_1, p_2, \ldots, p_N \right\} \]
\[ \{ R \} = \left\{ \frac{r_1}{w(r_1) \Delta r}, \frac{r_2}{w(r_2) \Delta r}, \ldots, \frac{r_N}{w(r_N) \Delta r} \right\} \]  

(1')

where \( w(r) \) is corresponding probability density function, e.g. Gaussian \( R \rightarrow N(r_m, \sigma_m) \), with

\[ M(R) = r_m = \frac{r_1 + r_N}{2}, \quad \Delta r = \frac{r_N - r_1}{N} = \frac{6 \sigma_m}{N} \]  

(2)

Therefore, Eq. (1) defines probabilities of appearance or existence of a cell with certain size.

III. CELLS AGGREGATION

The essence of proposed model are elementary aggregates, so-called cell molecules, composed of different size cells, which are considered as basic building units of all network. The aggregate structure depends on probability of cell [6], [7].

Here we also introduce the principle of minimum which means structure compactness. Such claim deserves careful explanation in order to avoid stiffness of the model. Two cells (as in Fig.1a) form, by all means, a dense structure.

\[ \text{Addition of the third cell already spoils previous compactness since cells are not solid impenetrable circles but rather circular emission areas which may easily overlap, even entirely. Therefore, among all possible cell dispositions we will take into account only those which do not overlap, as it is shown on Fig.1b. In other words, we consider extremal cases.} \]

The probability of cell aggregate with certain structure is given as

\[ P(i, j, \ldots, q) = \prod_{s=1}^{N} P_i \prod_{j \neq s} P_j \]  

(3)

where we differ central and surrounding cells (Fig.1b). Different cells sizes yield structural versatility of aggregate and the number of cells \( s \) in it. The central cell may be any one from the set (1), therefore, according to Eq. (3), \( i = 1,2, \ldots, N \), that means \( r_1 = r_1, r_2, \ldots, r_N \).

Also, the first of surrounding cells can be any from (1), i.e. \( j = 1,2, \ldots, N \), or \( r_j = r_1, r_2, \ldots, r_N \), and the same holds for the next one, \( k = 1,2, \ldots, N \), \( r_k = r_1, r_2, \ldots, r_N \). However, the third one may depend on the preceding two, which is determined by the ratio \( r_3/r_1 \). Such relationships can be described by functions

\[ j = f_\beta(i), \quad k = f_\eta(j) = f_\eta(i), \ldots \]  

(4)

where the functions \( f_\beta \) and \( f_\eta \) are given for the sake of completeness.

Nature and form of function set (4) will be clearly described in the case of an aggregate composed of the same size cells, i.e.

\[ f_\beta = f_\eta = \cdots = f_{\eta(q-1)} = 1 \]  

(5)

Whether probabilities in (3) are simple or conditional? According to (4) they are conditional. Theoretically, insertion of the last cell depends on sequence of previously arranged ones. Lower-order functions in (4) are mostly of formal significance. Cell size versatility means high independence in the beginning of aggregation process. Hence, probabilities in (3) can be considered as simple, because surrounding cells takes their place around central one quite independently in the beginning, except the last one or last two. That highly simplifies simulation. The total number of many different structures of aggregates cannot be determined by combinatorial analysis because involving elements does not possess equal significance.

As we already mentioned, we consider extremal aggregates, compact but non-overlapping dispositions of cells. The aggregation takes place according to the minimum principle, meaning compactness of aggregate structure, where surrounding cells are tightly fitted around central one, without gaps in an ideal case. This condition holds with accuracy of \( \Delta r \) (1), because of its discrete nature, and could be expressed by a criterion of full perimeter

\[ 2 \pi - \Delta \alpha = \sum_{n=1}^{N} \alpha_n - \sum_{n=1}^{N} \delta \alpha_n \leq 2 \pi \]  

(6)

Meaning of labels in (6) is explained on Fig.1c, where

\[ \sin \alpha_n = \frac{r_n}{r_c + r_n} \quad \text{or} \quad \alpha_n = \arcsin \left( \frac{r_n}{r_c + r_n} \right) \]  

(7)

Fig.2. Measure of angle compactness of cell aggregate.

The label \( r_c \) stands for radius of central cell, which is surrounded by other ones. Angles \( \delta \alpha_n \), that depend on the ratio of radii of neighbouring cells, can be neglected for simplicity. Value of \( \Delta \alpha \) represents deviation of a particular aggregate structure from ideal compactness, with \( \Delta \alpha = 0 \).

All structural variations spread between some typical configurations. For the given range of cell radii \([r_{min}, r_{max}]\) we point out the following ones.
The structure \((1+6)\), composed of same size cells, \(r_c = r_n, \quad n = 1,2,\ldots,6\), \(r_c = r_i + i \cdot \Delta r, \quad i = 1,2,\ldots,N - 1\).

Next to the previous structure is minimally incompact aggregate, \((1+6-\delta)\) (Fig.2), in which cell radii are minimally different \(r_c = r_i, \quad r_n = r_{i(n)} = r_i - \Delta r, \quad \forall n, \forall i\) (Fig.2), and we can write

\[
\frac{(2\pi - \Delta r^2) - 2\pi}{2\pi} = [C] = s - 1 = 6 \tag{8}
\]

It is easy now to determine the most probable of all structures and their mathematical expectation as well.

\[
M(I, J, K,..., Q) = \sum_{i,j,k,...,q} A(i,j,k,...,q) \cdot P(i,j,k,...,q) \tag{9}
\]

IV. AGGREGATION OF CELL AGGREGATES

Model of aggregate enables us to describe more complex network as a system of elementary aggregates (Fig.3a) in accordance to compactness principle. That means compact overlapping tailoring of aggregates [7].

Therefore, we proceed with next modeling level and represent the aggregate by an equivalent cell.

The first parameter is probability of existence or appearance of such equivalent cell, as an integral expression of its structure through probabilities of all cells within an aggregate.

Equivalent cell should have certain geometry. Some forms of aggregates are presented above giving an idea of all possible structure variations. There we recognize general structures of circle, ellipse, rounded rectangle etc. Modeling of a network with such versatile granular geometries would be extremely complicated and would cancel all benefits taken from aggregate replacement by equivalent granule.

Therefore, it is necessary to represent all aggregate structures by an unique and the most universal equivalent form, for which, after careful examination, we choose ellipse of various eccentricity.

The elliptic cell should encompass average peripheral profile of aggregate having interaction with similar neighboring equivalent cells. So, relying on aggregate structure main centroids [8] of equivalent ellipse should be determined. The first reference in that process is gravity center of aggregate figure, i.e.

\[
x_c = \frac{\sum_j S_j x_j}{\sum_j S_j}, \quad y_c = \frac{\sum_j S_j y_j}{\sum_j S_j} \tag{10}
\]

where \(S_j\) is the surface of circular cell within aggregate.

To avoid complicated variational methods we propose approach similar to calculation of planar moment of inertia. We easily obtain

\[
I_x = \sum_i f_i^{(0)} + x_c^2 \cdot \sum_i S_i, \quad I_y = \sum_i f_i^{(0)} + y_c^2 \cdot \sum_i S_i, \tag{11}
\]

\[
I_{xy} = \sum_i f_i^{(0)} + x_c y_c \cdot \sum_i S_i
\]

The angle of principal axis follows directly

\[
tg 2\phi_0 = -\frac{2I_{xy}}{I_x - I_y} \tag{12}
\]

In that way the principal axis of equivalent ellipse is obtained. It does not means ellipse of inertia but only the axis along which equivalent ellipse will be placed. The next step is to determine lengths of its both axes, or the equivalent ellipse shape, that would adequately express interactivity. Interaction itself, in this model, takes place at the point. Outer points of peripheral cells in aggregate are more prominent and exposed to possible contacts with neighboring aggregates than inner ones, i.e. those situated in depressions between granules (Fig.3b).

Such interactive difference of perimeter points can be geometrically expressed by fitting additional cells around given aggregate (Fig.2b). Their radius needs to represent a statistical average, since all peripheral cells within neighboring aggregates can make contacts. Therefore, the best particular value will be mathematical expectation of radius which embeds statistical average in the procedure of modeling equivalent cell. Circular segments create so-called contour of interaction. This one is to be approximated by equivalent ellipse as smoothly as possible (Fig.3b). Mathematical details of such extremal problem will not be considered in this paper.

Sets of possible aggregate structures and elliptical equivalent are correspondent, although the last one is considerably less dispersive, because of smoothing effect. That makes an opportunity to classify entirely range of elliptical cells into several representative ones. Since the shape of an ellipse is described by the ratio of its two principal axes, and expresses an equivalent of aggregate shape, classification should preserve those features. In other words, integral parameters such as surface or perimeter are not appropriate for intended clustering. Otherwise, too much structural information could be lost. Therefore, classification is performed according to axes, as a matrix quantization

\[
\{G_k\} \leftrightarrow \begin{bmatrix} a_1 b_1 & a_1 b_2 & \cdots & a_1 b_u \\ \vdots & \vdots & \ddots & \vdots \\ a_t b_1 & a_t b_2 & \cdots & a_t b_u \end{bmatrix} \tag{13}
\]

where

\[
\{a\} \xrightarrow{\text{scaling}} a, \quad \forall a \in [a_i - \Delta a/2, a_i + \Delta a/2] \Rightarrow a = a_i \tag{14}
\]

V. ROLE OF ROUTING NODES

So far we treated our model of network as composed of impenetrable circles, each with equal significance for the network. If the central cell is treated as switching node the model remains unchanged from the aspect creating equivalent cell.

Quite another story begins when one of peripheral cells, or corresponding nodes, i.e. mobile devices, takes role of...
router towards neighbouring cell aggregate(s). In that case the meaning of mechanical equivalence of cell aggregate becomes insignificant [6].

Therefore, we must propose another approach, which would reflect much more the very functional features of cell aggregate.

In that regard we will distinguish central or switching node and routing node. There can be one or several routing nodes depending on the network complexity and position of an aggregate within cell cluster. When functional aspects of an aggregate prevail geometrical representation will be quite different than before so it may be expressed by a vector (Fig.3)

\[
\begin{align*}
S^{(a)} & \rightarrow R^{(s)}_k \\
& \iff S^{(a)}(x^{(a)}_C, y^{(a)}_C) \rightarrow R^{(s)}_k(x^{(a)}_C, y^{(a)}_C)
\end{align*}
\]

\[n = 1, 2, ..., N_i; \quad k = 1, 2, ..., K(n)\] (15)

As opposed to LANs where geometrical aspects play negligible role, since only topology of connections are important to define network structure, in ad hoc networks geometry is much more crucial, despite the fact that the two mobile stations would be connected just if they are within their mutual radio reach, no matter their exact positions. But when routing capabilities take their place positions of mobile devices become important.

In that case the whole aggregate can be reduced in geometrical sense to two or three nodes with distances between them taken as respective vectors. The idea of such geometrical equivalence is given on Fig.3. There can be seen that the switching, i.e. central node perform also routing functions while peripheral ones only serves as routers, or connections to neighbouring aggregates. Here we see two distinct cases or types of routing segments in the whole route, labeled as

\[
S^{(a)}R^{(s)}, \quad S^{(a)}R^{(s)}_j, \quad i = j = 1, 2, ..., K(1)/K(N) (16)
\]

which stands for ending segments, and

\[
R^{(s)}_iS^{(a)}R^{(s)}_j; \quad i = j = 1, 2, ..., K(n); \quad i \neq j; \quad n = 1, 2, ..., N
\] (17)

for mediating ones.

Here we introduced somehow unusual mathematical labels for vectors – conventional ones for ending segments, as true ones, and two-sided arrow for system of two vectors between two routers over central switch within an aggregate (Fig.4).

Therefore, the whole network, no matter how complex, can be represented as a chain of vectors, representing the elements of the route across engaged cell aggregates. It is simply expressed as

\[
AR = S^{(a)}R^{(s)}_i + \sum_{n=1}^{N-1} R^{(s)}_iS^{(a)}R^{(s)}_j + S^{(a)}R^{(s)}_j (18)
\]

\[i = j = 1, 2, ..., K(1)/K(N)\]

Elements of ad hoc networks are of highly unpredictable behaviour. Because of that fact the Eq. (18) can drastically change its structure. Here we will not consider temporal aspects of presented model. Our intention was just to give an outline of proposal.

VI. CONCLUSIONS

Ad hoc networks are very specific entities for research because of their stochastic nature. While simple in structure modeling of their establishing, functioning and disassembling is not so difficult. Getting much powerful in processing aspect mobile devices, the basic building blocks of ad hoc networks, enabled very complex network structures which are difficult to model and simulate. Therefore, certain simplifications are quite necessary.

In this paper we proposed simplifying equivalents of ad hoc networks in two ways.

At first, we gave a geometrical equivalent of first-order cell aggregate, having the central cell surrounded by densely packed and disposed outer cells. Such compact dispositions are obviously extremal cases of aggregate appearance but make the modeling much easier. Finally, it gives us a referent indication for real dispositions.

Secondly, if we consider much complex ad hoc networks, consisted of many partially overlapped aggregates, where each of them possess internal switching and routing capabilities, we think it is of vital importance to outthink certain effective simplified representation which would describe the whole network structure in geometrical, or parametric way. There we proposed vector-chain description that expresses routing aspects while keeping basic geometrical structure of the network.

We estimate that the both proposed approaches carry significant simulating potential and perspective for further development.

REFERENCES


